

Authentic Learning Activities in Middle School Mathematics:

Data Analysis,
Statistics,
& Probability

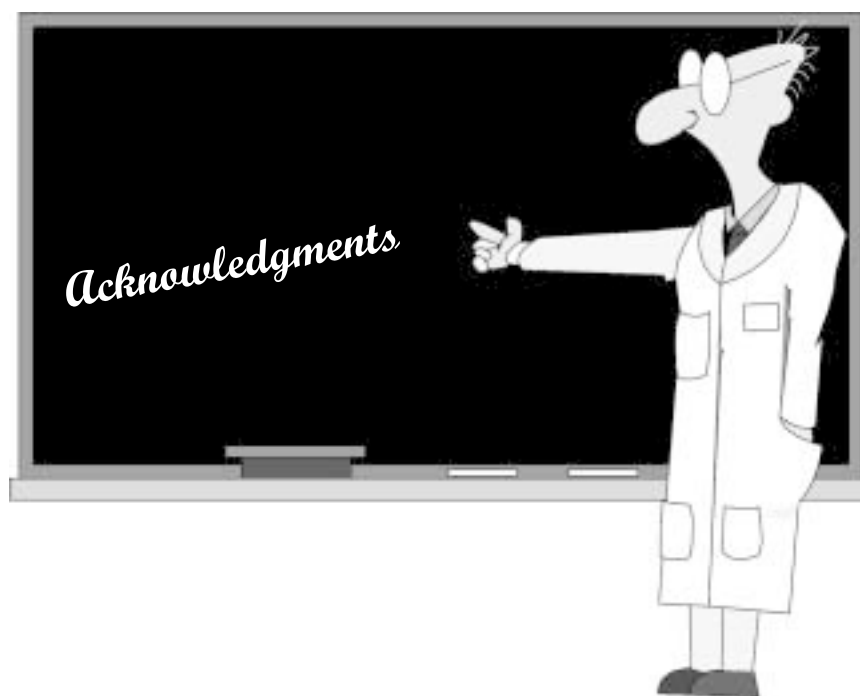
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<http://www.edu.gov.on.ca>



This module is the final product in a series of drafts, revisions and field tests conducted during the 1997-98 school year. Enhancing the usefulness of this booklet is the plethora of wonderful samples of student work that appear under the headings “WHAT YOU MIGHT SEE”. For these samples we are deeply indebted to the grade 7 and 8 students of the following teachers:

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INTRODUCTION TO THE MODULES

The *Ontario Curriculum, Grades 1-8: Mathematics* issued in 1997, has redefined the elementary school mathematics curriculum for Ontario. New expectations for student learning require the teaching of new mathematical topics as well as a shift in emphasis of content previously taught. In particular, the new document reflects the growing need for students to expand their skills in processing information, managing data, problem solving, and using technology to achieve these ends. While there is a reduced attention to rehearsing rote skills such as, long division with large divisors, or extraction of roots by the formal method, there is a reaffirmation of the need for students to master the multiplication tables and fundamental pencil-and-paper skills that underpin arithmetic facility. Such skills are intended to support the intelligent use of technology in performing complex computations of the type that arise in so-called “real world” contexts.

Implicit in this document is the demand for new or revised methods of instruction and assessment. Educational research of the past twenty years has mounted a compelling argument for a knowledge-building approach to instruction (See page 9), that reduces the role of the teacher as purveyor of information and enhances the teacher’s role as facilitator of learning. With this shift in instructional methodology comes a corresponding demand for change in methods of assessment. (See page 10).

The call for such changes in curriculum, instruction, and assessment has created a need for teachers of grades seven and eight to plan new programs in mathematics from the plethora of print and electronic resources currently available. Since most of these teachers are responsible for many subject areas in addition to mathematics, the consolidation of these materials into a set of coherent lessons is daunting. To support teachers in this quest, the Ministry of Education and Training has commissioned a set of five modules, (of which this is one) that gather together many of the extant resources in a single reference package. Each module addresses one of the five strands in the new curriculum.

Though they address different content strands, all modules have the same format. Part I outlines the rationale underpinning the ideas and activities developed in the module. Part II provides a brief instruction for teachers on the new content in that strand. Part III provides for grade 7 teachers, a set of sample activities that constitute a unit of instruction. This unit is intended to model the instructional and assessment philosophies discussed in Part I. It is not intended to cover the entire content of the strand, or to replace any resources presently used, but rather to supplement the current program. Publishers have written correlations linking their print materials to the sample unit, and these are included as an insert so they can be updated periodically. Included in Part III under the heading, “What You Might Expect to See” are samples of student work, classified by achievement level, and presented opposite a rubric that will help you assess the work of your students. Part IV parallels Part III, except it is keyed to the grade 8 unit. However, it is recommended that all teachers familiarize themselves with the contents of both Parts III and IV. Part IV concludes with a selected list of appropriate print and media resources at the grade 7-8 levels and some useful Internet addresses to fulfill the intent that the module provide a single reference to help teachers implement the new curriculum.

THE RATIONALE FOR DATA MANAGEMENT

STATISTICS

The rationale for the inclusion of statistics in the Ontario mathematics curriculum is given on page 61 of *The Ontario Curriculum, Grades 1-8: Mathematics*.

The related topics of probability and statistics are highly relevant to everyday life. Graphs and statistics bombard the public in advertising, opinion polls, reliability estimates, population trends, descriptions of discoveries by scientists, estimates of health risks, and analyses of students' performance in schools and schools' performance within school systems, to name just a few.

This rationale supports the following statement from the *Curriculum and Evaluation Standards for School Mathematics* published by the National Council of Teachers of Mathematics* in 1989.

Students [in grades 5 to 8] need to be actively involved in each of the steps that comprise statistics, from gathering information to communicating results. Identifying the range or average of a set of data, constructing simple graphs, and reading data points as answers to specific questions are important activities, but they reflect only a very narrow aspect of statistics. Instead, instruction in statistics should focus on the active involvement of students in the entire process: formulating key questions; collecting and organizing data; representing the data using graphs, ... Students' understanding of statistics is also enhanced by evaluating others' arguments.

The unit on the African Elephant presented in Part III of this module attempts to embed the foregoing elements in motivational activities that address the expectations of the new curriculum.

PROBABILITY

Also on page 61 of *The Ontario Curriculum, Grades 1-8: Mathematics*, we find the following rationale for the inclusion of probability in the new curriculum.

Students should actively explore situations by experimenting with and simulating a variety of probability models. The focus should be on real-world questions - such as the probable outcome of a sports event or whether it will rain on the day of the school picnic. Students should talk about their ideas and use the results of their experiments to model situations or predict events. The topic of probability is rich in interesting problems that can fascinate students and provide bridges to other topics, such as ratios, fractions, percents, and decimals.

In this statement, the new curriculum endorses the following recommendation on page 110 of *Curriculum and Evaluation Standards for School Mathematics*.

In modeling problems, conducting simulations, and collecting, graphing, and studying data, students will come to understand how predictions can be based on data. Mathematically derived probabilities can be determined by building a table or tree diagram Students develop an appreciation of the power of simulation and experimentation by comparing experimental results to the mathematically derived probabilities.

The unit titled "Is the World Series Rigged?" in Part IV develops probability concepts in a way that incorporates the spirit and intent of these statements.

*The National Council of Teachers of Mathematics (NCTM) is the largest world-wide organization of mathematics educators, with a membership exceeding 110 000. It plays a major leadership role in setting new visions for mathematics education.

NEW TECHNOLOGIES & THEIR IMPLICATIONS FOR GRADES 7 & 8

The policy on the use of technology is embodied in *The Ontario Curriculum, Grades 1-8: Mathematics* is stated on page 7 of that document.

Students are expected to use calculators or computers to perform operations that are lengthier or more complex than those covered by the pencil-and-paper expectations. When students use calculators and computers to perform operations, they are expected to apply their mental computation and estimation skills in predicting and checking answers. Students will also use calculators and computers in various experimental ways to explore number patterns and to extend problem solving.

The rationale for this policy is clearly expressed on page 30 of the *National Council of Teachers of Mathematics 1997-98 Handbook*.

Technology is changing the ways in which mathematics is used and is driving the creation of new fields of mathematical study. Consequently, the content of mathematics programs and the methods by which mathematics is taught and learning assessed are changing. The ability of teachers to use the tools of technology to develop, enhance, and expand students' understanding of mathematics is crucial. These tools include computers appropriate calculators (scientific, graphing, programmable, etc.), videodisks, CD-ROM, telecommunications networks by which to access and share real-time data, and other emerging educational technologies. Exploration of the perspectives these tools provide on a wide variety of topics is required by teachers.

It is the position of the National Council of Teachers of Mathematics that the use of the tools of technology is integral to the learning and teaching of mathematics. Continual improvement is needed in mathematics curricula, instructional and assessment methods, access to hardware and software, and teacher education.

In describing implications of this policy at the classroom level, the 1997-98 Handbook provides, on page 6, more detail pertaining to grade 7 and 8 classes

A comprehensive mathematics curriculum should help students learn to use calculators, computers, and other technological tools. These tools are used in many aspects of their education and influence how they study mathematics, science, or engineering in college or how they will do mathematics in the workplace. It would be remiss not to make their use a part of contemporary mathematics education.

The Curriculum and Evaluation Standards for School Mathematics makes it clear, however, that such tools "do not replace the need to learn basic facts, to compute mentally, or to do reasonable paper-and-pencil computation" (p. 19). The Standards also suggest that when used appropriately, calculators and computers enable students to explore new areas of mathematics and to tackle many challenging mathematical problems that are impractical to attempt without the aid of such tools. Indeed, calculators and computers with appropriate software can transform the classroom into a laboratory where students can investigate and experiment with mathematical ideas.

UNDERSTANDING THE LEARNING PROCESS & ITS IMPACT ON INSTRUCTION

In this and the other four modules, we present activities that attempt to incorporate a range of instructional approaches including traditional instruction. The students are sometimes given information and required to read, interpret and apply it in an exercise. In other cases, the students must investigate, explore and discover concepts that lurk beneath the surface of an activity. In some cases, the students will work individually, while in others they will work collaboratively or cooperatively. Often an activity is dedicated to the resolution of a question that generates an intellectual dilemma or conflict and stimulates mathematical thinking. The activity in Part III of this module asks whether the banning of the sale of ivory is an effective strategy for preserving the African Elephant. The activity in Part IV asks students to determine whether the World Series is “rigged” to make it last 7 games.

In view of these multiple perspectives on how children learn, one might assume that all traditional approaches to teaching will disappear as these new philosophies are incorporated. However a response to the question “What should I see in a [NCTM] Standards-based mathematics classroom?” the *NCTM 1997-98 Handbook* presents a balanced and accessible image of effective instruction:

First and foremost, you'll see students doing mathematics. But you'll see more than just students completing worksheets. You'll see students interact with one another, use other resources along with textbooks, apply mathematics to real-world problems, and develop strategies to solve complex problems.

Teachers still teach. The teacher will pose problems, ask questions that build on students' thinking, and encourage students to explore different solutions. The classroom will have various mathematical and technological tools (such as calculators, computers, and math manipulatives) available for students to use when appropriate. The teacher may move among the students to understand their thinking and how it is reflected in their work, often challenging them to engage in deeper mathematical thinking.

ASSESSMENT: RUBRICS & ACHIEVEMENT LEVELS

The changes in curriculum and instruction described on the preceeding pages have significant implications for assessment and evaluation. Among these implications is the shift from norm-referenced to criterion-referenced assessment, as described on page 1 of *The Assessment Standards for School Mathematics* published by the NCTM in 1995.

At present, a new approach to assessment is evolving in many schools and classrooms. Instead of assuming that the purpose of assessment is to rank students on a particular trait, the new approach assumes that high public expectations can be set that every student can strive for and achieve, that different performances can and will meet agreed-on expectations, and that teachers can be fair and consistent judges of diverse student performances.

The Ontario Curriculum, Grades 1-8: Mathematics, (See pp. 4-5) also embraces the move to criterion-referenced assessment, and includes four levels of achievement for describing student performance.

High achievement is the goal for all students, and teachers, students, and parents need to work together to help students meet the expectations specified. The achievement levels are brief descriptions of four possible levels of student achievement. These descriptions, which are used along with more traditional indicators like letter grades and percentage marks, are among a number of tools that teachers will use to assess students' learning. The achievement levels for mathematics focus on four categories of skills: problem solving, understanding of concepts, application of mathematical procedures, and communication of required knowledge. When teachers use the achievement levels in reporting to parents and speaking with students, they can discuss with them what is required for students to achieve the expectations set for their grade.

Descriptions of the four levels of achievement for problem solving, concepts, applications, and communication are shown on page 9 of that document. These are the levels for concept understanding.

knowledge/skills	Level 1	Level 2	Level 3	Level 4
Understanding of concepts	The student shows understanding of concepts:			
	– with assistance	– independently	– independently	– independently
	–by giving partially complete but inappropriate explanations	–by giving appropriate but incomplete explanations	– by giving both appropriate and complete explanations	– by giving both appropriate and complete explanations, and by showing that he or she can apply the concepts in a variety of contexts
	–using only a few of the required concepts	–using more than half the required concepts	–using most of the required concepts	– using all of the required concepts

A table such as the one above that describes levels of achievement is called a *rubric*. Included with the student activities in this and the other modules, are rubrics and samples of student work that exemplify the levels of student performance as defined in *The Ontario Curriculum, Grades 1-8: Mathematics*.



PART II

New Content in Data Management & Probability

STEM-AND-LEAF PLOTS

Activity 3 of the grade 7 sample unit (see p. 40) is titled “The Ivory Wars”. In that activity, students are presented with this list of masses (in kilograms) of 40 tusks found in a caché of ivory. They are to find the median mass.

14	22	16	9	8	13	21	6
36	17	20	5	35	36	22	16
31	16	23	13	3	12	8	10
20	35	14	22	3	36	37	7
8	10	31	3	17	11	8	3

One approach would be to attempt to order the masses from least to greatest, but this requires scanning the entire data set for the smallest item. Upon removal of the smallest item from the list, it is necessary to repeat the entire process with the remaining masses to obtain the second smallest item, and so on.... To avoid such an inefficient and tedious procedure, we could proceed to make a table to group the numbers in decades as follows:

Masses of the Tusks in Kilograms

numbers between 0 and 9	3	3	3	3	5	6	7	8	8	8	8	9		
numbers between 10 and 19	10	10	11	12	13	13	14	14	16	16	16	17	17	
numbers between 20 and 29	20	20	21	22	22	22	23							
numbers between 30 and 39	31	31	35	35	36	36	36	37						

Since all the numbers in the same row have the same first digit, it is redundant to record it, and we need only record the ones digit. (The tens digit of a number in the first row is assumed to be 0, the tens digit of a number in the second row is assumed to be 1, and so on.) In this abbreviated form, our table of numbers looks like this:

Stem-and-Leaf Diagram for Masses of the Tusks

tens	ones
0	3 3 3 3 5 6 7 8 8 8 8 9
1	0 0 1 2 3 3 4 4 6 6 6 7 7
2	0 0 1 2 2 2 3
3	1 1 5 5 6 6 6 7

Since the tens digit can be regarded as the stem, and the units digits are the “sprouts” or “leaves” on this stem, the display above is called a *stem-and-leaf* plot.

The stem-and-leaf plot is a useful visual display for the calculation of the mean, median and mode of a set of data. But first, let's review our definitions of central tendency.

The *mean* of a set of numbers is obtained by dividing the sum of those numbers by the number of numbers; for example, the mean of the set of 5 numbers 4, 7, 13, 17, 19 is:

$$(4 + 7 + 13 + 17 + 19) \div 5 \quad \text{or} \quad 12.$$

The *median* of a set of numbers is obtained by arranging the numbers in order and choosing the middle number; for example, the median of the set of 5 numbers 4, 7, 13, 17, 19 is 13.

When there are an even number of numbers, there are two middle numbers, so the median is obtained by taking the mean of the two middle numbers. For example, the median of the set of numbers 4, 7, 13, 17, 19, 20 is: $(13 + 17) \div 2$ or 15.

The *mode* of a set of numbers is the number that occurs most often in the data set. In the list of numbers 3, 4, 6, 6, 6, 7, 7, 9, 9, the mode is 6 because it occurs most often.

We are now in a position to obtain the mean, median and mode of the masses of elephant tusks directly from our stem-and-leaf plot.

Calculation of the mode: The stem-and-leaf display shows that there are four 3's and four 8's in our data set. No other number has occurred four times, so our data is bimodal; that is, 3 and 8 are modes.

Calculation of the median: Since there are 40 numbers in our data set, the median is the mean of the two middle numbers, the 20th and the 20st in size. These are obtained by counting along the stem-and-leaf display to the 20th number. It is 14. The 21st number is 16, so the median is $(14 + 16) \div 2$ or 15.

Calculation of the mean: A conventional way to find the mean would be to find the sum of all forty numbers and then divide by 40. When we have the numbers arranged in a stem-and-leaf array, there is an easier way. The stem-and-leaf shows there are 13 numbers with tens digit 1, 7 numbers with tens digit 2, and 8 numbers with tens digit 3. Altogether, the tens digits represent a total of $13 \times 10 + 7 \times 20 + 8 \times 30 = 510$. The units digits total 167, so the sum of all the numbers is $510 + 167 = 677$. The mean is $677 \div 40 = 16.925$.

By the way, there is one additional advantage to the stem-and-leaf diagram that you might share with your students. When you rotate the stem-and-leaf plot 90°, it becomes a histogram showing the number of numbers that fall in each decade ... hmmm...interesting!



ones		tens	
0	3 3 3 3 5 6 7 8 8 8 9	0	
1	0 1 2 3 3 4 4 6 6 6 7 7	1	
2	0 0 1 2 2 2 3	2	
3	1 1 5 5 6 6 6 7	3	

SPREADSHEETS & DATA BASES



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A spreadsheet is an invaluable mathematical tool, but, as Sherman has discovered, its power is limited to the insight of the user. In essence a spreadsheet is just a rectangular array of cells. Each cell has an address or location identified by the row and column in which it is located. Often the row is specified by a number and the column by a letter, so M7 is the address of the cell in column M of row 7. The power of the spreadsheet lies in its ability to relate the quantity in any particular cell by a mathematical relationship or formula to the value in any other cell. Therefore, if the cells in row 7 represent the retail prices of a series of items, and the cells in row 8 represent the final prices after the 8% Ontario sales tax is added, then the numbers in row 8 would be linked to the numbers in row 7 through multiplication by 1.08. When the retail prices in row 7 are changed, the new final prices are automatically recalculated! This remarkable feature enables us to investigate “what if” scenarios. When we change the input in a procedure, the spreadsheet automatically calculates all the new values and displays the final output.

In the exercises on page 32, students are asked to use the computer to construct bar and circle graphs showing the elephant populations in various regions of Africa. To do this they will use a spreadsheet program in its simplest form, entering the names of the regions of Africa into one row or column of a spreadsheet and the populations by region for a given year into another column or row. Upon selection of the appropriate type of graph by the student, the spreadsheet will display the required graph. On page 41 of the grade 7 Activity titled, *The Ivory Wars*, students use a stem-and-leaf plot to calculate the mean and median of the masses of 40 elephant tusks found in a caché. In this activity, students use a stem-and-leaf plot to find the mean and median masses. As an exercise in building formulas on a spreadsheet, the students could enter all 40 masses into a row or column of a spreadsheet and define a cell to contain the sum of all the 40 numbers in that row, divided by 40. This cell would display the mean mass. Students could explore how changing one or more masses would change the mean mass. This is a dynamic way to help students see how sensitive the mean is to the inclusion of extreme values. In the other modules in the Impact Math Series, you will see additional ideas for incorporating spreadsheets into your instructional activities.

While spreadsheets are rectangular arrays into which you enter and manipulate information, databases are sources of information and data often displayed as rectangular arrays of information. As the Internet expands, it includes an ever expanding menu of databases which students can access to learn data management in real contexts. One of the best databases available to Canadian students is that provided by Statistics Canada. The best way to learn it is to use it, so we have provided you with its URL and its home page on page 15.

SPREADSHEETS & DATA BASES

Statistics Canada is accessible on the Internet at this website:

<http://www.statcan.ca>

Have students access this page and explore the Daily news, the Canada Quiz, the Education Resources, and above all, the Canadian Statistics sites. This is an invaluable resource that can provide data for real life mathematics investigations. One of the activities offered by Statistics Canada under the Education Resources is, “What is the average height in your class?”, an on-line exercise that explores the concepts of central tendency. The Statistics Canada home page is shown below.



Welcome to Statistics Canada!

▶ Daily news

Highlights of newly released data, schedules for major releases and announcements of new products and services.

▶ 1996 Census

Free tabular data from the latest Census results.

▶ Education Resources

Programs and products to integrate Canadian statistical information into teaching and learning

▶ Service centres

Information on our regional offices, libraries and other distributors of Statistics Canada data.

▶ Site Map

▶ Links to other sites

▶ Français

▶ Canadian Statistics

Free tabular data on aspects of Canada's economy, land, people and government

▶ Products and services

Catalogue; research papers; online statistics (\$); downloadable publications (\$); information on seminars, conferences and other services.

▶ Concepts, definitions and methods

Program documentation; standard classifications; discussion papers; questionnaires; information on new surveys.

▶ About Statistics Canada

▶ The Talon Bulletin & Guide

▶ Canada Quiz

Explore our site and learn about Canada and Canadians.



COMPARATIVE BAR CHARTS: DOUBLE & STACKED

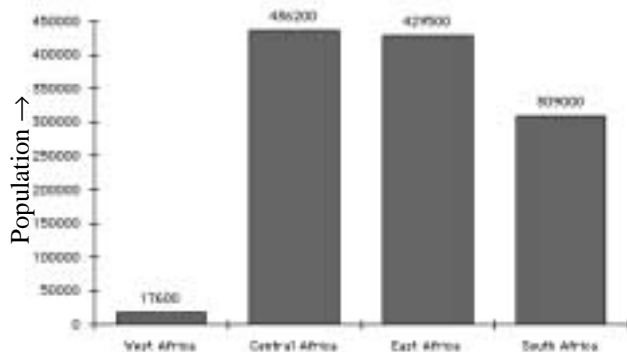
Activity 2 of the grade 7 sample unit is titled *Elephant Populations in the 1980's* (see p. 30). In that activity, students are given the following information showing the elephant populations by region in 1981 and 1989.

Elephant Populations in 1981	
West Africa	17 600
East Africa	429 500
Central Africa	436 200
Southern Africa	309 000

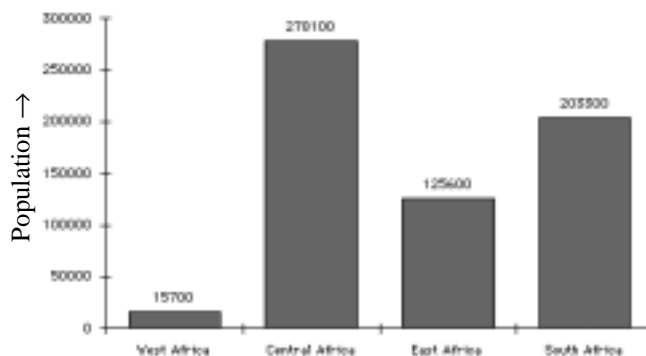
Elephant Populations in 1989	
West Africa	15 700
East Africa	125 600
Central Africa	278 100
Southern Africa	203 300

They are asked to show this data on bar graphs, first using grid paper, and then using a computer. When creating graphs on grid paper, some students find difficulty choosing an appropriate scale. They often forget to label the graphs appropriately. However, when using a spreadsheet, most students are successful in creating graphs like these.

Elephant Populations in 1981



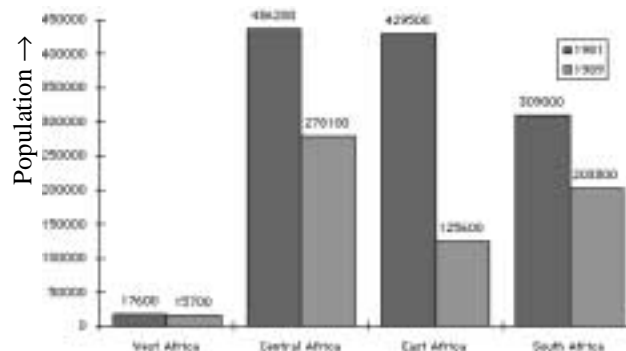
Elephant Populations in 1989



Note: It is important to remind students that the horizontal axis of a bar graph is usually a discrete set of things such as, “West Africa”, “East Africa” etc. and so there are spaces between the bars. However, when the heights of the bars in a graph represent frequencies and the horizontal axis is a continuous scale, as in the *World Series simulation* on page 71, there is no space between the bars and the graph is called a *histogram*.

In the elephant population activity, students soon discover that it is easier to see the decline of the elephant population in each region by combining the two bar graphs into a single *comparative bar graph* such as the one shown here, called a *double bar graph*. An alternative kind of comparative bar graph is the so-called *stacked graph* that places one bar on top of the other. In the elephant population example, a stacked graph would provide a good visual display of the population decline because there was a population decline in all of the regions. However, if some regions showed increases and others a decrease, the stacked graph can be confusing.

Elephant Populations in 1981 & 1989



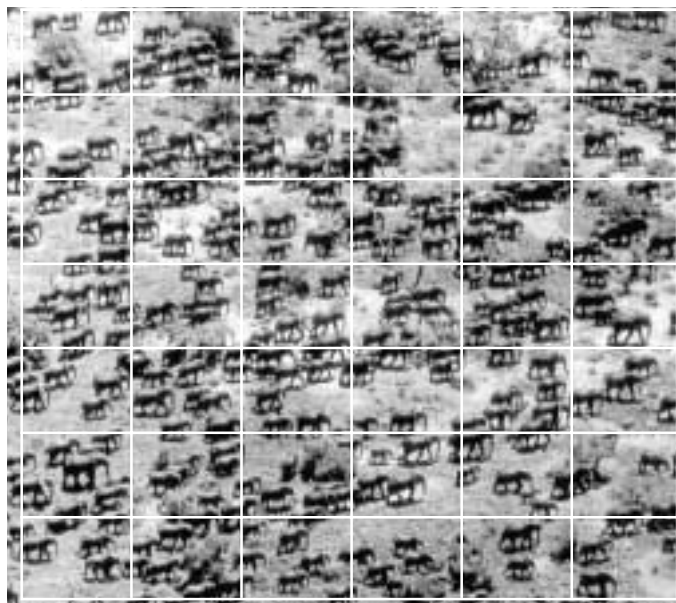
SAMPLING CONCEPTS: RANDOM SAMPLES & BIAS

The Ontario Curriculum, Grades 1-8: Mathematics contain the following expectations aimed at helping students understand the importance of avoiding bias when sampling from a population and identifying bias in reports based on samples.

DM 7 - 11 *By the end of grade 7, students will analyse bias in data collection methods.*

DM 8 - 11 *By the end of grade 8, students will assess bias in data collection methods.*

These expectations require that students understand how to make inferences about a collection of things, called a *population*, from a sample of that population. (Note that the term “population” as it is used in statistics is not necessarily a collection of people or animals. We could attempt to determine whether 40% of all M&M’s in the world (a population) are brown by selecting a *sample* of 1000 and determining the proportion of the M&M’s in the sample that are brown.)



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In Activity 1 of our grade 7 unit, (See p. 25.) the students are shown a grid of 42 rectangles superimposed on a herd of elephants, and asked to select a sample of 10 rectangles and count all the elephants in those 10 rectangles. Since the students are to determine the mean number of elephants per rectangle and use this to estimate the herd size, it is important that they take a representative sample, i.e. some rectangles with few elephants and some with many. Students are asked to explain how they chose the rectangles and it is hoped that they will demonstrate an awareness of the need to choose either a representative sample or a random sample.

Another aspect of sampling that students need to understand is that small samples are less likely to be representative of a population than large samples. Students might be asked, “Why not base

your estimate of the herd size on the number of elephants in a single rectangle?” The importance of using large samples, where possible, can be further developed in this activity by pooling the total count of all elephants from all students and observing how close the class estimate of the herd size is to the actual herd size. (The actual herd size can be obtained by having each group count all the elephants in each row of the grid.)

Ultimately it is hoped that students will emerge from this activity and others that you provide, with an understanding that sampling is more reliable as a predictor of population characteristics when:

- the sample is randomly selected
- the sample is large.

Students should be exposed to a wide range of ways that samples can be biased (See list of current texts, p. 96) and asked to identify the flaws in various sampling techniques.

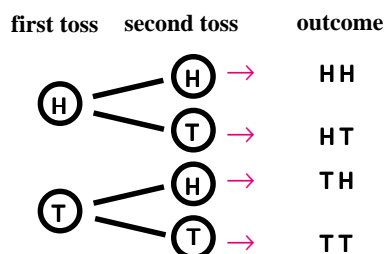
THEORETICAL & EXPERIMENTAL PROBABILITY

THEORETICAL PROBABILITY

The Theory of Probability is based on the idea that the likelihood of any outcome from the toss of a coin or the roll of a die can be calculated in terms of a set of equally likely outcomes. For example, on the roll of a fair die all 6 numbers from 1 to 6 have an equally likely chance of turning up. Consequently, we assign a probability of $1/6$ to each outcome. Therefore the probability of an “event” such as “the number that turns up will be greater than four” is obtained by listing the number of outcomes that can produce (*are favourable to*) that event. Since the outcomes 5 and 6 yield a number greater than four, then the probability that a number greater than four appears on the toss of a fair die is $2/6$. More formally, we define the (*theoretical*) probability of an event A as follows:

$$\text{Probability of event A} = \frac{\text{Number of outcomes favourable to A}}{\text{Number of possible (equally likely) outcomes}}$$

When a coin is tossed twice, there are four equally likely outcomes possible. We denote them by HH (two heads), HT (head followed by a tail), TH (tail followed by a head) and TT (two tails). To display all these possible outcomes, we use a *tree diagram*.



In Activity 3 of the grade 8 sample unit, (See pp. 78-79) we help students build tree diagrams for the toss of a coin 3 and 4 times. The students are then provided with a template that helps them build the tree diagram for the toss of a coin 5 times. They use this tree diagram to calculate the probabilities of some relatively complex events by merely counting outcomes on the tree diagram.

EXPERIMENTAL PROBABILITY & SIMULATIONS

Few events in real life contexts can be assigned a probability in the sense defined above. The probability of rain or snow cannot be expressed in terms of equally likely outcomes. In the grade 8 sample unit, we pose the question, “Is the World Series rigged?”. The World Series of baseball is a best 4 out of 7 series that has gone the full 7 games almost 25 of the last 50 years. Some have suggested that it is rigged by the owners to maximize their revenue from ticket sales. We cannot determine precisely the probability that a World Series will go 7 games, because there are too many variables. However, if we assume the teams are evenly matched, we can look on the outcome of a game as the toss of a coin. We can then *simulate* a World Series outcome by tossing a coin until a total of 4 heads or 4 tails occurs and then recording the frequency of times the best 4 out of 7 went a full 7 games. By dividing by the number of trials, we obtain the *relative frequency* of the 7-game outcome. In Activity 2 of this unit, students toss coins to run such a simulation and obtain a relative frequency or *experimental probability* to help them decide whether the World Series may be “rigged”.



PART III

Data Management in Grade 7

THE ONTARIO CURRICULUM, GRADES 1-8: MATHEMATICS

DATA MANAGEMENT AND PROBABILITY: GRADE 7

Overall Expectations

By the end of Grade 7, students will:

- systematically collect, organize, and analyse data;
- recognize the different levels of data collection;
- use computer applications to examine and interpret data in a variety of ways;
- develop an appreciation for statistical methods as powerful means of decision making;
- construct graphic organizers using computer applications;
- interpret displays of data and present the information using mathematical terms;
- evaluate data and make conclusions from the analysis of data;
- use and apply a knowledge of probability.

Specific Expectations

(For convenient reference, the specific expectations are coded. DM 7-1 refers to the first data management expectation in grade 7. P 7-3 denotes the third probability expectation in grade 7.)

Students will:

Collecting and Organizing Data

- | | |
|--------|---|
| DM 7-1 | - demonstrate the pervasive use of data and probability; |
| DM 7-2 | - understand the impact that statistical methods have on decision making; |
| DM 7-3 | - collect and organize data on tally charts and stem-and-leaf plots, and display data on frequency tables, using simple data collected by the students (primary data) and more complex data collected by someone else (secondary data); |
| DM 7-4 | - understand how tally charts and frequency tables can be used to record data; |
| DM 7-5 | - understand the difference between a spreadsheet and a database for recording and retrieving information; |
| DM 7-6 | - search databases for information and interpret the numerical data; |

Analysing Data

- DM 7-7 - understand that each measure of central tendency (mean, median, mode) gives different information about the data;
- DM 7-8 - identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes (e.g., line graphs that level off);
- DM 7-9 - describe in their own words information presented on tally charts, stem-and-leaf plots, and frequency tables.
- DM 7-10 - use conventional symbols, titles, and labels when displaying data;
- DM 7-11 - analyse bias in data-collection methods;
- DM 7-12 - read and report information about data presented on bar graphs, pictographs, and circle graphs, and use the information to solve problems;
- DM 7-13 - describe data using calculations of mean, median, and mode;

Concluding & Reporting

- DM 7-14 - display data on bar graphs, pictographs, and circle graphs, with and without the help of technology;
- DM 7-15 - make inferences and convincing arguments that are based on data analysis (e.g., use census information to predict whether the population in Canada will increase);
- DM 7-16 - evaluate arguments that are based on data analysis;
- DM 7-17 - explore with technology to find the best presentation of data

Probability

- P 7-1 - develop intuitive concepts of probability and understand how probability can relate to sports and games of chance;
- P 7-2 - list the possible outcomes of simple experiments by using tree diagrams, modelling, and lists;
- P 7-3 - identify the favourable outcomes among the total number of possible outcomes and state the associated probability (e.g., of getting a heads in a fair coin toss);
- P 7-4 - apply a knowledge of probability in sports and games of chance.

ACTIVITY 1 – TEACHER EDITION

WILL THE AFRICAN ELEPHANT BECOME EXTINCT IN YOUR LIFETIME?

Expectations Addressed

- DM 7-1 • demonstrate the pervasive use of data
- DM 7-2 • understand the impact that statistical methods have on decision making
- DM 7-13 • describe data using a calculation of the mean
- DM 7-15 • make inferences that are based on data analysis
- DM 7-16 • evaluate arguments that are based on data analysis

Context

Organizations such as the World Wild Life Foundation have identified over 150 species that are threatened with extinction. and have created several classifications to indicate how close a species is to extinction. The designation “endangered” is assigned to a species whose population is so small, or whose habitat is so damaged, that it will soon be extinct if measures are not taken to protect it. The classification “threatened” is assigned to a species which has several subspecies, at least one of which is endangered. The issue of endangered species is an important one for students, since it will be in their adult lives that the consequences of present measures will be felt.

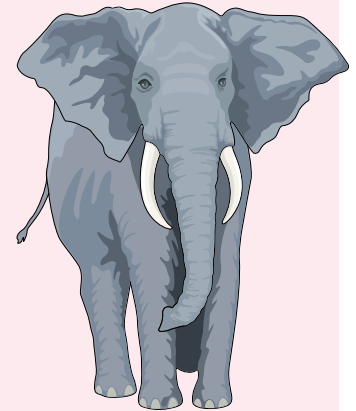
In Activity 2, (See p. 32) we present data that shows the depletion of the Elephant populations by region between 1981 and 1989. These populations were estimated by counting the number of members of herds seen from the air. More recently, satellites are used to track the movements of elephant herds and to ascertain the numbers of members in these herds.

In 1990, the African elephant, world’s largest land mammal, was placed on Appendix I of the Convention on International Trade in Endangered Species (acronym CITES). With this came a ban on the trade of ivory in most countries. This ivory ban has reduced substantially the rampant killing of the elephants and bodes well for their survival. In June 1996, the African elephant status was upgraded from endangered to threatened, and some limited trading of ivory was permitted for those three countries with an abundance of elephants. Whether banning the ivory trade will be beneficial or detrimental to the survival of the elephant remains an issue of considerable controversy.

ACTIVITY 1 – STUDENT PAGE

WILL THE AFRICAN ELEPHANT BECOME EXTINCT IN YOUR LIFETIME?

The African elephant is the world’s largest land mammal. Males can weigh up to 7 tonnes and stand 4 m tall at the shoulder. It consumes between 150 and 250 kg of vegetation and 160 L of water every day. Its average lifespan is between 60 and 75 years.



Sadly, this magnificent creature may become extinct during your lifetime. That means there may be no more elephants in the wild forty or fifty years from now.

There are three main reasons why the elephant populations are decreasing:

1. They have been killed in great numbers for their ivory tusks.
2. The fast-growing African human population needs more land to grow food and is moving into the elephant territory and destroying their habitat.
3. The Sahara desert is expanding into areas that once grew trees on which elephants could feed, but now those trees are gone.

In 1989 CITES, a UN sponsored organization, placed the African Elephant on the *endangered species list*, indicating that the species is in danger of extinction. This classification prohibited international sales of ivory, discouraging the slaughter of elephants for their tusks. When it appeared that poaching was under control in 1990 the African elephant status was changed from *endangered* to *threatened*. This meant that the African elephant was endangered in some parts of Africa but not in all areas.

Determining areas where the elephant is endangered requires accurate estimates of the elephant populations. But how is this achieved? Elephants that roam the plains of Africa are photographed from aircraft. The aerial photo is used to count the elephants in each herd and estimates are compiled from these samples. Those elephants that live in the mountainous areas and dense forests are most difficult to count, because they cannot be seen from the air. Instead, scientists cut paths through the forests and then count dung piles and compute from these, estimates of the elephant populations. Precise estimates of elephant populations are further complicated by the fact that elephants migrate distances up to 800 km and so there is a possibility that the same herd may be counted twice as it travels across borders. Estimating elephant populations though challenging, is vitally important to the survival of these wonderful animals!

ACTIVITY 1 – TEACHER EDITION

The Lesson Launch

5 minutes

This lesson can be launched by asking:

What is the largest land animal in the world?

Once the elephant has been identified by the students as world's largest land animal, invite students to tell what they know about elephants, including physical features, dimensions and eating habits. Have students identify the elephant's dimensions relative to their own. That is, the elephant is about twice the height of an adult human and about 50 times the mass. Explain that there are two species of elephants, the African and the Asian. Discuss the differences in both species, noting that the African elephant is the larger, but is not trainable like its Asian cousin. Mention that the African elephant has been listed as an endangered species and ask students what that means. It is important that students emerge from this discussion realizing the significance of the elephant and that its survival is threatened.

Initiating Activity

10 minutes

Hand out the student copy of the 2-page spread, *The Extinction of World's Largest Land Mammal*. Choose a student to read the first two paragraphs to the class.

Then select another student to read the three main causes of the decreasing elephant population. Pause to discuss these causes with the class, ensuring that students understand how the sale of ivory and the burgeoning African population both jeopardize the survival of the elephant.

A third student should read the next paragraph, and this should be followed with a brief explanation of the difference between the classifications of *endangered* and *threatened*. Have one of the better readers in the class read the final paragraph. (There will be some student laughter when the "dung piles" are mentioned, so dig right in and discuss this method in earnest – suggest that it might be a way to count dogs in the neighbourhood.) Follow with a brief discussion of the methods of estimating the elephant populations.

Small Group Activity

25 minutes

Divide students into pairs to discuss and complete the answers to questions ❶ through ❹. Allow students who finish ahead of the others to attempt exercise ❺. When most students have completed the first 6 exercises in their notebooks, explain that exercise ❻ requires them to count the number of elephants in ten different rectangles and find the mean. Invite the students to explain how they can use this mean to estimate the number of elephants in the herd. Allow them five minutes to complete this.

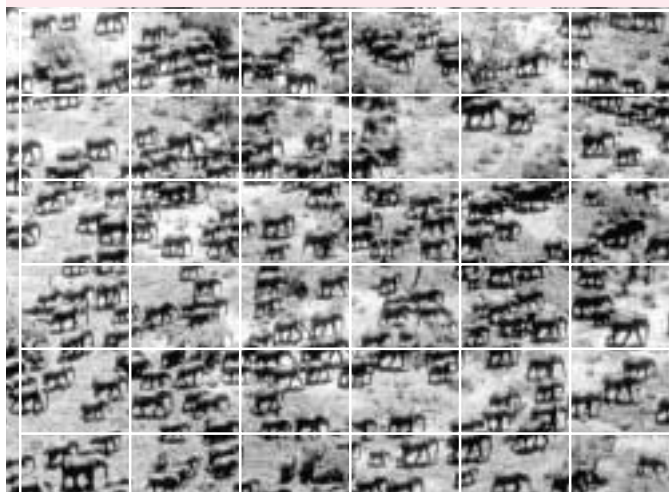
Closure

When all the students have finished, invite each group to write their estimated number of elephants on the blackboard. Then ask the students to use their calculators to determine the mean of all the estimates. Compare this mean with the actual number and discuss why averaging all their estimates is likely to give a very good estimate.

ACTIVITY 1 – STUDENT PAGE

WILL THE AFRICAN ELEPHANT BECOME EXTINCT IN YOUR LIFETIME?

The figure below shows a grid overlay on an aerial photo of a herd of elephants. In the exercises you will discover how this is used to estimate their number.



- ❶. How would you distinguish between the terms "endangered species", and "threatened species"?
- ❷. What are the main causes for the decline in the elephant population.
- ❸. What cause do you think is the most important? Explain.
- ❹. What do you think could be done to save the elephant from extinction?
- ❺. What information do you think is needed to determine whether the elephant is near extinction?
- ❻. Describe how scientists gather this information. What difficulties do they encounter in making accurate estimates? How do they solve these problems?
- ❼. Choose ten of the rectangles at random. Count and record the number of elephants in each rectangle. Calculate the mean number of elephants in those rectangles. Estimate the number of elephants in the herd.

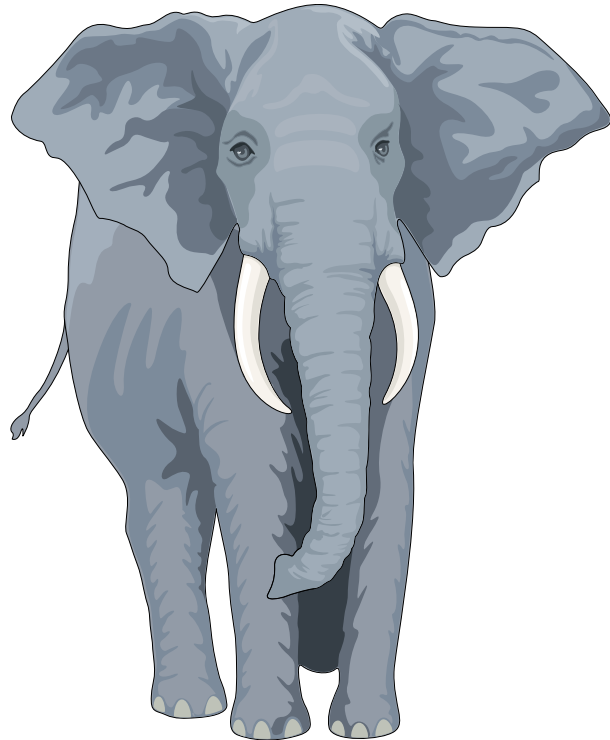
WILL THE AFRICAN ELEPHANT BECOME EXTINCT IN YOUR LIFETIME?

The African elephant is the world's largest land mammal. Males can weigh up to 7 tonnes and stand 4 m tall at the shoulder. It consumes between 150 and 250 kg of vegetation and 160 L of water every day. Its average lifespan is between 60 and 75 years.

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3. The Sahara desert is expanding into areas that once grew trees on which elephants could feed, but now those trees are gone.



In 1989 CITES, a UN sponsored organization, placed the African Elephant on the *endangered species list*, indicating that the species is in danger of extinction. This classification prohibited international sales of ivory, discouraging the slaughter of elephants for their tusks. When it appeared that poaching was under control in 1990 the African elephant status was changed from *endangered* to *threatened*. This meant that the African elephant was endangered in some parts of Africa but not in all areas.

Determining areas where the elephant is endangered requires accurate estimates of the elephant populations. But how is this achieved? Elephants that roam the plains of Africa are photographed from aircraft. The aerial photo is used to count the elephants in each herd and estimates are compiled from these samples. Those elephants that live in the mountainous areas and dense forests are most difficult to count, because they cannot be seen from the air. Instead, scientists cut paths through the forests and then count dung piles and compute from these, estimates of the elephant populations. Precise estimates of elephant populations are further complicated by the fact that elephants migrate distances up to 800 km and so there is a possibility that the same herd may be counted twice as it travels across borders. Estimating elephant populations though challenging, is vitally important to the survival of these wonderful animals!

ACTIVITY 1 – STUDENT ACTIVITY PAGE

WILL THE AFRICAN ELEPHANT BECOME EXTINCT IN YOUR LIFETIME?

The figure below shows a grid superimposed on an aerial photo of a herd of elephants. In the exercises you will discover how this is used to estimate their number.



©Peter Beard / THE IMAGE BANK

- ❶. How would you distinguish between the terms “endangered species”, and “*threatened species*”?
- ❷. What are the main causes for the decline in the elephant population?
- ❸. What cause do you think is the most important? Explain.
- ❹. What do you think could be done to save the elephant from extinction?
- ❺. What information do you think is needed to determine whether the elephant is near extinction?
- ❻. Describe how scientists gather this information. What difficulties do they encounter in making accurate estimates? How do they solve these problems?
- ❼. Choose ten of the rectangles at random. Explain how you chose these rectangles. Count and record the number of elephants in each rectangle. Calculate the mean number of elephants in those rectangles. Estimate the number of elephants in the herd (including the hidden ones).

GRADE 7
ANSWER KEY FOR ACTIVITY 1

- ❶. **Endangered Species:** Animals whose numbers are so low, or whose habitat has been so badly destroyed, that they will become extinct if nothing is done.

Threatened Species: A species that has several subspecies – some in danger and some not.

- ❷. The three main reasons why the elephant populations are decreasing:

- They have been killed in great numbers for their ivory tusks.
- The fast-growing African human population needs more land to grow food and is moving into the elephant territory and destroying their habitat.
- The Sahara desert is expanding into areas that once grew trees on which elephants could feed, but now those trees are gone.

- ❸. Answers will vary, but the student response should provide some support for why that particular cause is identified as the *most* important of the three.

- ❹. Excellent answers will involve something more creative than merely, “Ban the sale of ivory”. Possibilities include creating elephant reserves, or irrigating the desert to expand the elephant habitat.

- ❺. To determine whether the elephant is near extinction, we need to have an estimate of its present population and of its population at a previous time, so we can determine whether its numbers are in decline.

- ❻. Complete answers will include at least the aerial photographs and the dung counts. The difficulties may include the problem in counting elephants from the air, and the danger in counting the same herd more than once.

- ❼. The student’s response should show:
- the number of elephants in each rectangle.
 - division by 10 to obtain the mean number of elephants per rectangle.
 - multiplication of this mean by 42 to obtain an estimate of the herd size.

Note: The bottom 12 rectangles of the grid on the elephant herd photo have been obscured to prevent the students from merely counting all the elephants, and to encourage them to make some assumptions about the densities of the elephants in those rectangles that they cannot see.

It is also important that the student’s answer shows some evidence of an attempt to select a representative sample of rectangles; i.e. some rectangles with few elephants and some with many. In some cases, students will identify and record the particular rectangles chosen.

The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 1				
	Level 1	Level 2	Level 3	Level 4
CONCEPTS				
Estimation from a Random Sample (exercise 7) DM 7-11, 7-15	<ul style="list-style-type: none"> •Estimation procedure is inappropriate. •No explanation is given. 	<ul style="list-style-type: none"> •Estimation procedure is inappropriate. •Estimate is given to an unwarranted level of accuracy. •Some attempt at explanation 	<ul style="list-style-type: none"> •Estimation procedure is appropriate. •Reasonable explanations are given. 	<ul style="list-style-type: none"> •High level of estimation skill. •Gives evidence of the need to choose rectangles randomly
COMMUNICATION				
Communication of required knowledge related to concepts, procedures, & problem solving (exercises 1 – 6)	<ul style="list-style-type: none"> •Responses are usually incomplete. 	<ul style="list-style-type: none"> •Responses sometimes incomplete. 	<ul style="list-style-type: none"> •Responses are mostly complete and usually include reasonable explanations 	<ul style="list-style-type: none"> •Responses are complete and clearly explained, showing creativity of content and expression.

ACHIEVEMENT LEVELS DEFINED BY THE MINISTRY OF EDUCATION AND TRAINING

- Level 1** Identifies achievement that falls much below the provincial standard.
- Level 2** Identifies achievement that approaches the standard.
- Level 3** The “provincial standard”, identifies a high level of achievement of provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.
- Level 4** Identifies achievement that surpasses the standard.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: ESTIMATION FROM A RANDOM SAMPLE

Level 1

The student has correctly determined the mean number of elephants in the ten rectangles selected. There is no evidence that the rectangles were selected randomly. The estimate of 32 elephants in the herd seems to suggest that the student is either unaware of the estimation procedure or what it means. It is even possible that the student had a different estimate and miscopied the answer. We'll never know, but there is no evidence of anything here beyond a Level 1 achievement.

Level 3

This student used an appropriate estimation procedure. She also explained the procedure carefully, showing an understanding of the need to choose the rectangles appropriately. She would have achieved a Level 4 rating if she had not expressed her estimate using an unwarranted level of precision (a decimal number). Also, her final sentence suggested that she thought she was estimating the average in "every herd".

Level 2

The student uses an inappropriate estimation procedure. There's no evidence that the student understands the need to take a random sample of rectangles.

Level 4

It doesn't get much better than this. The student was clever enough to show which rectangles he chose and the numbers showed that he chose sparsely and densely populated rectangles. He also explained the procedure carefully, showing an understanding of the need to document the process.

WHAT YOU MIGHT SEE

COMMUNICATION OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 1

The student answers are incomplete or terse with little explanation. There is no evidence of anything here beyond a Level 1 achievement.

Level 3

This student understands that we need to know the elephant populations at two distinct times in order to determine whether there is a population decline. Also the student suggest two ways to preserve the elephant populations.

Level 2

The answers are sometimes complete, but the answer to exercise 5 does not indicate that we need to know the elephant populations at two different times to determine whether it is decreasing. Furthermore, the answer to exercise 6 mentions only one way that scientists count elephants.

Level 4

This student communicates well her understanding of the difficulty in controlling the growth of human population. She also expresses herself well in her suggestions for preserving the elephant populations and in suggesting different kinds of problems scientists might face in counting elephants.

ACTIVITY 2 – TEACHER EDITION

ELEPHANT POPULATIONS IN THE 1980's

Expectations Addressed

- DM 7-1 • demonstrate the pervasive use of data
- DM 7-2 • understand the impact that statistical methods have on decision making
- DM 7-5 • understand the difference between a spreadsheet and a data base for recording information
- DM 7-8 • identify and describe trends in graphs, using informal language to identify growth, clustering, and simple attributes.
- DM 7-10 • use conventional symbols, titles, and labels when displaying data.
- DM 7-14 • display data on bar graphs, pictographs, and circle graphs, and use the information to solve problems.
- DM 7-15 • make inferences that are based on data analysis
- DM 7-17 • explore with technology to find the best presentation of data.

Context

In Activity 1, students were introduced to the declining elephant populations and the potential extinction of this great mammal. In this activity, we increase the level of sophistication of student understanding of the problem by providing the populations of elephants in each of the four major regions of Africa in 1981 and 1989. Students plot these data for 1981 and 1989, on separate bar graphs, and then in a comparative bar graph to discover that the declines in the elephant population vary significantly from region to region. These graphs are plotted first on grid paper. (For the grid paper black line master, see page 56.) Then the graphs are plotted using a computer spreadsheet. (When these activities were field-tested, a very high percentage of students favoured graphing using the technology, because, in their words, “It was easier”, and “It was *funner*”.

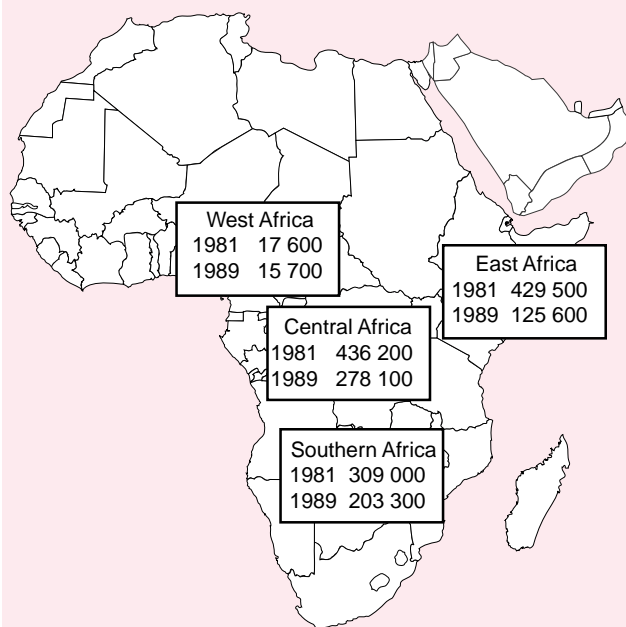
The composition of the four regions by country is given in the table on the right. These data can be used as a basis for a more in-depth study of the elephant population decline by country.

Students are then prompted to use a computer spreadsheet to construct circle graphs showing the distribution of the elephant population across the four regions in 1981 and again in 1989. When asked to determine which graph, the double bar graph, or the circle graph is more appropriate for measuring population decline, students are prompted to reflect on the meaning of each type of graph. Most students correctly identify the double bar graph, but fewer can express why.

ACTIVITY 2 – STUDENT PAGE

ELEPHANT POPULATIONS IN THE 1980's

This map shows the elephant populations in four regions of Africa in 1981 and in 1989.



ELEPHANT POPULATIONS		
Country	1981	1989
Zaire	376 000	103 000
Central African Republic	31 000	27 000
Congo	10 800	25 000
Gabon	13 400	92 000
Central Africa- Total	436 200	278 100
Kenya	65 000	18 000
Tanzania	203 900	75 000
Sudan	133 700	21 000
East Africa – Total	429 500	125 600
Botswana	20 000	58 000
South Africa	8 000	8 200
Zambia	160 000	45 000
Zimbabwe	47 000	49 000
Southern Africa – Total	309 000	203 300
West Africa – Total	17 600	15 700
Africa – Total	1 192 300	622 700

ACTIVITY 2 – TEACHER EDITION

The Lesson Launch

5 minutes

This lesson could follow a geography lesson in which a map of Africa was presented and in which students were asked to identify as many countries as they can on that continent. After a discussion of the locations of the Sahara Desert, the jungle regions, and the Savannah plains, students could be led into a discussion of the elephant populations with a series of questions such as the following.

•Do you think the population of elephants in the Sahara Desert region declined very much between 1981 and 1989? If not, why not?

•In what kinds of regions, deserts, plains or jungles do you think the elephant populations declined the most? Explain.

Students will offer a variety of conjectures, some valid and some not, but the important thing is to hook their interest, and to have them understand that measuring a decline in population requires that we measure the population at two different times.

Paired Activity

40 minutes

Hand out the student copy of the 2-page spread, *Elephant Populations in the 1980's*. Have each student choose a partner, and provide 3 copies of the grid paper (See black line master p. 56.) to each pair of students. Assign exercises 1 to 5 to the students and indicate that each pair is to cooperate in producing a final report. Allow those who finish first to continue with exercises 6 through 9 on the computer.

When most students have finished the grid paper exercises, call them together to discuss the graphs and remind them to correctly label all graphs and to provide clear explanations to exercises 6 and 7. Some students may require instruction on using spreadsheets to construct graphs on computer. If there are not enough computers to accommodate all the pairs, assign to those who are waiting the elephant problem at the bottom of page 33.

Closure

When all the students have finished, discuss their findings. Ask questions such as the following to clarify the concepts of percent and percentage decline:

- Does “the region with the greatest population decline” always mean “the region in which there is the greatest percentage decline in population”?
- If the elephant population of a region declined 30% between 1981 and 1989, what is its population in 1989 as a percent of its population in 1981?
- If the elephant population declines another 30% between 1989 and 1997, what is the 1997 population as a percent of its population in 1981?

ACTIVITY 2 – STUDENT PAGE

Make a table to show the population of elephants in 1981 and 1989 in each of the four regions of Africa.

How many rows will you need in your table?

How many columns?

On grid paper:

1. Make a bar graph to show how many elephants there were in 1981 for each of the four regions in Africa.
2. Label both axes of your graph. Give your graph a title that tells what it's about.
3. Make a bar graph like the one in 1 but for 1989.
4. Make a new bar graph that has two bars beside each other for each region in Africa. The bar on the left should show the elephant population in 1981 and the bar on the right should show the population in 1989.
5. The graph you created in 4 is called a *double bar graph*. Label the axes of your double bar graph and give it a name.

6. Describe what happened to the elephant populations between 1981 and 1989. In what region was the decline the greatest? the least?

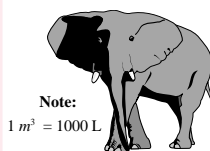
7. a) In what region did the elephant population suffer the greatest percentage decrease between 1981 and 1989? Explain how you know.

b) In what region did the elephant population decrease by about 36%?

8. Use a computer to create bar graphs showing the elephant populations in each region of Africa in 1981 and 1989. Describe some advantages and disadvantages in using a computer to construct graphs.

9. Enter into a spreadsheet the elephant populations of each of the four regions in Africa in 1981. Use the chart menu to make a circle graph showing what percent of the total elephant population comes from each region. Repeat this for 1989. Which graph, the circle or double bar graph do you think is better for showing the decline in the elephant populations in the 1980's? Explain.

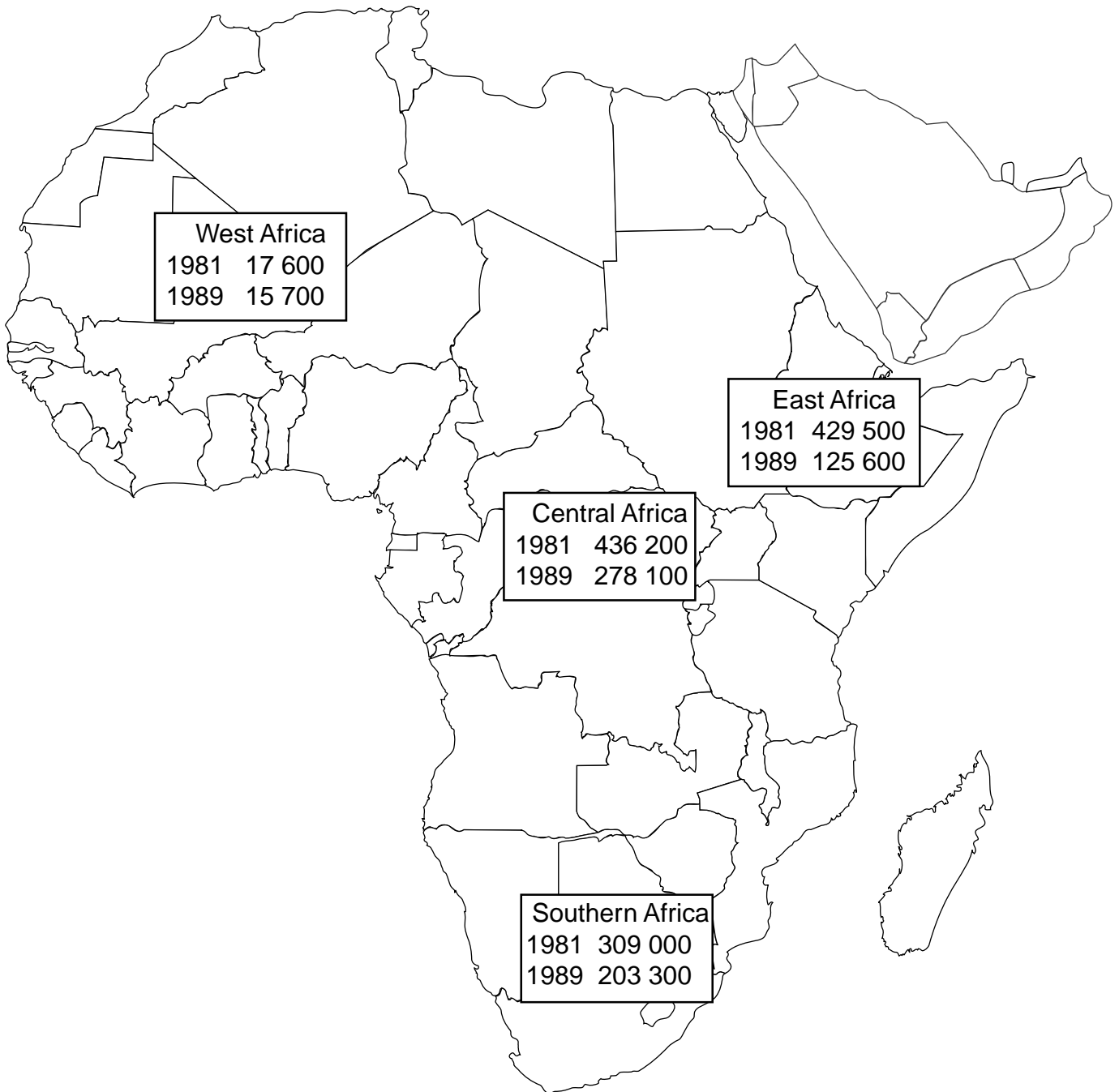
PROBLEM



An adult elephant consumes about 160 L of water per day. How else could you report this information? About how many days could a herd of 60 adult elephants survive on a rectangular trough full of water if the trough measures 5 m × 2 m × 2 m?

ELEPHANT POPULATIONS IN THE 1980's

This map shows the elephant populations in four regions of Africa in 1981 and in 1989.



ACTIVITY 2 – STUDENT ACTIVITY PAGE

ELEPHANT POPULATIONS IN THE 1980's

Make a table to show the population of elephants in 1981 and 1989 in each of the four regions of Africa.

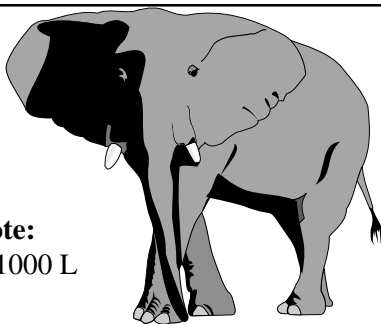
How many rows will you need in your table?

How many columns?

On grid paper:

- ➊. Make a bar graph to show how many elephants there were in 1981 for each of the four regions in Africa.
 - ➋. Label both axes of your graph. Give your graph a title that tells what it's about.
 - ➌. Make a bar graph like the one in ➊ but for 1989.
 - ➍. Make a new bar graph that has two bars beside each other for each region in Africa. The bar on the left should show the elephant population in 1981 and the bar on the right should show the population in 1989.
 - ➎. The graph you created in ➍ is called a *double bar graph*. Label the axes of your double bar graph and give it a name.
- ➏. Describe what happened to the elephant populations between 1981 and 1989. In what region was the decline the greatest? the least?
 - ➐. a) In what region did the elephant population suffer the greatest percentage decrease between 1981 and 1989? Explain how you know.
b) In what region did the elephant population decrease by about 36%?
 - ➑. Use a computer to create bar graphs showing the elephant populations in each region of Africa in 1981 and 1989. Do you prefer to use a computer? Describe some advantages and disadvantages in using a computer to construct graphs.
 - ➒. Enter into a spreadsheet the elephant populations of each of the four regions in Africa in 1981. Use the chart menu to make a circle graph showing what percent of the total elephant population comes from each region. Repeat this for 1989. Which graph, the circle or double bar graph do you think is better for showing the decline in the elephant populations in the 1980's? Explain.

Note:
 $1 \text{ m}^3 = 1000 \text{ L}$



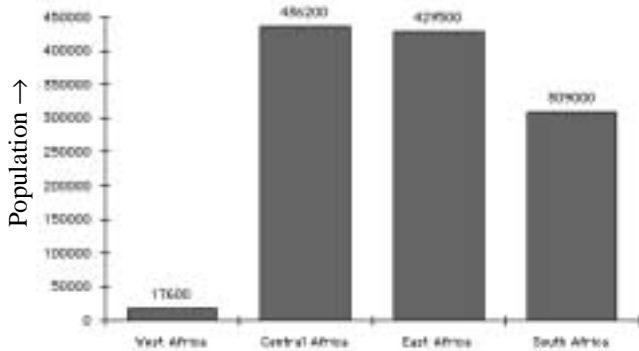
PROBLEM

An adult elephant consumes about 160 L of water per day. How else could you report this information? About how many days could a herd of 60 adult elephants survive on a rectangular trough full of water if the trough measures $5 \text{ m} \times 2 \text{ m} \times 2 \text{ m}$?

GRADE 7

ANSWER KEY FOR ACTIVITY 2

① & ② Elephant Populations in 1981

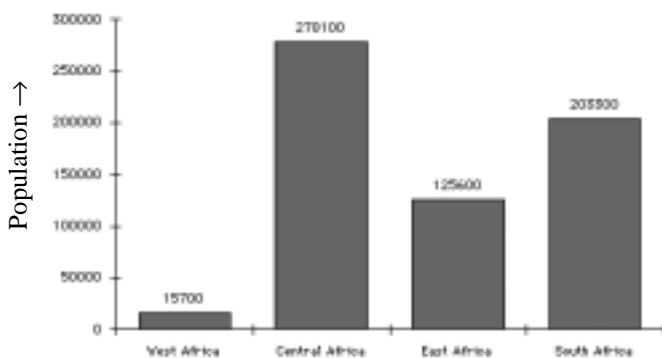


⑥ The populations of elephants decreased substantially in all regions of Africa between 1981 and 1989.

The decline was greatest in East Africa;
i.e., $429\,500 - 125\,600 = 303\,900$.

The decline was least in West Africa;
i.e., $17\,600 - 15\,700 = 1900$

③ Elephant Populations in 1989

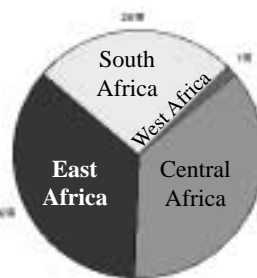


⑦ a) The population of elephants decreased by $303\,900 \div 429\,500$ or about 70% in East Africa between 1981 and 1989.

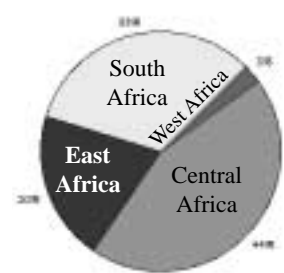
b) The decline in Central Africa between 1981 and 1989 was about
 $(436\,200 - 278\,100) \div 436\,200$
or about 36.2%.

⑨ The circle graphs for 1981 and 1989 are:

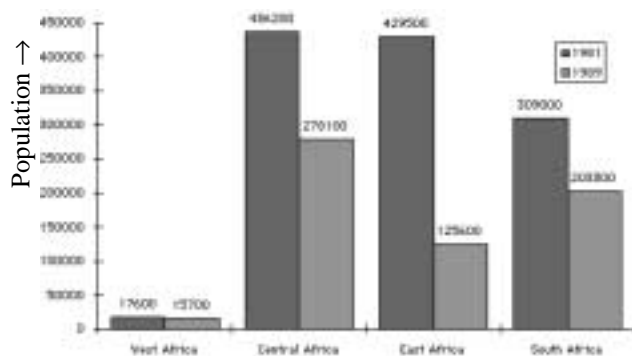
Elephant Populations in 1981



Elephant Populations in 1989



④ & ⑤ Elephant Populations in 1981 & 1989



The circle graphs show only how the elephants are distributed among the four regions of Africa. Only the bar graphs enable us to determine the population decline from 1981 to 1989.

PROBLEM

- The trough holds 20 000 L
- 60 elephants consume $60 \times 160 \text{ L} = 9600 \text{ L}$ per day.
- Therefore, 60 elephants could survive for $20\,000 \div 9600$ or 2.08 days. That is, 60 elephants could survive for about 2 days.

The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 2				
	Level 1	Level 2	Level 3	Level 4
APPLICATION				
Constructed Bar Graphs (exercises ❶ – ❺) DM 7-14	<ul style="list-style-type: none"> •All or most graphs are incomplete 	<ul style="list-style-type: none"> •Some graphs are incomplete 	<ul style="list-style-type: none"> •Most graphs are complete and appropriately labelled. 	<ul style="list-style-type: none"> •All bar graphs are complete and appropriately labelled.
CONCEPTS				
Interpretation of Graphs (exercises ❸, ❹ & ❺) DM 7-8, 7-17	<ul style="list-style-type: none"> •Incorrect interpretation of graphs. 	<ul style="list-style-type: none"> •Some interpretations of the bar graphs are incorrect. 	<ul style="list-style-type: none"> •Most interpretations of the bar graphs are correct. •Some evidence of the understanding that the circle graphs do not show population declines. 	<ul style="list-style-type: none"> •The interpretations of the bar graphs are correct. •Evidence of the understanding that the circle graphs do not show population declines.
APPLICATION				
Constructed Circle Graphs (exercise ❹) DM 7-5, 7-14	<ul style="list-style-type: none"> •Incorrect circle graphs. 	<ul style="list-style-type: none"> •Some circle graphs are incomplete. 	<ul style="list-style-type: none"> •Most circle graphs are complete. 	<ul style="list-style-type: none"> •All circle graphs are correctly labelled and well presented.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: CONSTRUCTED GRAPHS

Level 1

The scales on the graphs are incorrect. For example, the 1989 elephant graph would suggest that the population of elephants in East Africa was 187 500, while the table gave this population as 125 600. Also, all graphs are missing labels on the vertical axis and the graph titles do not indicate what is plotted.

Level 3

This graph has been constructed correctly, except it is missing a label for Southern Africa.

This graph is constructed correctly and labelled correctly, but there should be a space between the bars for the different regions of Africa.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: CONSTRUCTED GRAPHS

Level 4

All these graphs are correctly constructed and clearly labelled. The percentages in the circle graph are shown and the sectors are labelled. There is no room for improvement.

UNDERSTANDING OF CONCEPTS: INTERPRETATION OF GRAPHS

Level 3

There is some evidence of the understanding that the circle graphs do not show population decline. However it is not clear whether the student is aware that the circle graph does not show population, but only population distribution.

Level 4

There is evidence of the understanding that the circle graphs show population distribution. The student indicates that the circle graph shows the *percentage* of the elephant population in each region.

ACTIVITY 3 – TEACHER EDITION

THE IVORY WARS

Expectations Addressed

- DM 7-1 • demonstrate the pervasive use of data
- DM 7-2 • understand the impact that statistical methods have on decision making
- DM 7-3 • collect and organize data on tally charts and stem-and-leaf plots, and display complex data collected by someone else (secondary data).
- DM 7-7 • understand that each measure of central tendency (mean, median, mode) gives different information about the data.
- DM 7-9 • describe in their own words information about data presented on tally charts, stem-and-leaf plots, and frequency tables.
- DM 7-13 • describe data using calculations of the mean and median.
- DM 7-15 • make inferences that are based on data analysis

Context

This activity directs student attention to one of the most controversial causes of the depletion of the elephant populations – the ivory trade. Introduction to the problem of the ivory poachers and the mass slaughter of families of elephants for the ivory in their tusks offends the students' deep humanitarian instincts. Their earliest understandings of the ivory trade usually incites an outrage that seeks resolution in the outright banning of the trade and the punitive destruction of all poachers. In this activity, we investigate the masses of 40 elephant tusks found in a hidden caché. The students organize these data into a stem-and-leaf plot and use this plot to determine the mean and median tusk masses. By examining the range of values and the extrema, the students observe that some of the tusks must belong to immature elephants, suggesting that the poachers have moved from the killing of large mature male elephants, to the slaughter of families. This hypothesis is further supported by the attendant articles that show how the mean mass of the tusks in the more recent reports seems to be diminishing. Students determine this by performing mathematical operations on the numbers given in the newspaper articles. (These are real data taken from actual articles on the Internet.) Throughout this activity, students are also prompted to consider carefully the difference in meaning of phrases such as “can weigh more than” and “the mean mass is”. The concept of central tendency is explored in linguistic as well as in mathematical terms. Once this activity is completed, the student is primed for a consideration of the other side of the problem – survival of the Africans.

ACTIVITY 3 – STUDENT PAGE

This sample of articles shows how the market for ivory (called “white gold”) caused poachers to think of elephants only in terms of the ivory in their tusks.

THE ECONOMIST July 1989

Saving the Elephant

The African Elephant like the blue whale and the black rhino is on a fast track to extinction. The African elephant's misfortune is its tusks [and] their main cash value as sources of ivory. ... The group at Imperial College reckon that 6828 tonnes of ivory were exported between 1979 and 1987. ... That suggests that between 680 000 and 760 000 elephants must have been killed during that period.

National Geographic May 1991

Shrinking Herds and Habitats

After shooting many of Africa's largest tuskers, poachers have turned their guns on younger elephants with smaller tusks, speeding the slaughter.

For each tonne of ivory, poachers shot nearly twice as many elephants in 1988 as in 1979.

African Wildlife News May - June 1997
<http://www.awf.org/news/ele.answers.html>

Although illegal killing has remained well below the pre-ban level [banning of the sales of ivory enacted in 1989], in some areas poaching has increased. At least 10 elephants in the Kenya-Tanzania border area have been killed in the last 15 months alone. In 1996 over 5 tonnes of ivory were discovered in Tanzania, possibly taken from as many as 1000 elephants conservation experts say.

ACTIVITY 3 – TEACHER EDITION

The Lesson Launch

5 minutes

To open this lesson, ask students if they recall the three main reasons for the depletion of the elephant populations. When a student mentions the ivory trade, explain that poachers used to hunt mainly the mature elephant bulls who had the largest tusks, but as the elephant populations declined, it was necessary to obtain ivory from any elephant that could be taken, including families. Ask students how we might determine from a caché of tusks, whether the poachers were killing younger elephants. Then ask students how we might determine from a caché found a few years later, whether the sizes of the elephants killed was changing. This discussion will lead naturally to the ideas of central tendency and the concept of average. This is a good opportunity to inject the concepts of the mean, the median and the mode as different measures of central tendency. Discuss the idea of extreme values in a set of data, and how these extreme values affect the mean and the median. Students should understand that these different measures of central tendency are all referred to as an “average”, but they have different meanings.

Individual Activity

25 minutes

Hand out the student copy of the 2-page spread, *The Ivory Wars*. Then review with the students the concept of a stem-and-leaf plot and enter the first few masses from the caché onto a stem-and-leaf plot and let the students proceed on their own to answer in their notebooks exercises ① and ②. Often students find it easier to create a stem-and-leaf plot on graph paper, placing a digit in each square. This way they obtain more even spacing in their graph and turning it 90° puts it into the familiar “histogram” format.

Place a copy of the *Guinness Book of Records* on the desk for easy access to students who want to research the mass of the heaviest tusk ever found.

Several students will have difficulty calculating the mean mass in exercise ②, because the *ECONOMIST* article gives two different estimates of the elephant populations. Explain to the students that these are the high and low estimates and the true number of elephants lies somewhere between. Explain that they are to find the high and low estimates of mass to define a range.

Closure

As the students complete exercise ②, some will come to realize that the mean mass of the elephant tusks is smaller in the later articles, suggesting that the poachers are harvesting increasingly younger elephants. Discuss this idea with the students and emphasize that this is an important example of how we use data to draw inferences.

ACTIVITY 3 – STUDENT PAGE

- ①. The conservation authorities discovered a hidden caché of 40 elephant tusks. These tusks probably came from the slaughter of a small herd. They recorded the mass of each tusk (shown below) to the nearest kilogram.

14	22	16	9	8	13	21	6
36	17	20	5	35	36	22	16
31	16	23	13	3	12	8	10
20	35	14	22	3	36	37	7
8	10	31	3	17	11	8	3

- a) Display the masses given above in a stem-and-leaf diagram. Would a newspaper use a stem-and-leaf to report data? Why or why not?
- b) How would you use the stem-and-leaf to find the masses of the heaviest and lightest tusks? Suggest reasons why there is such a difference in the sizes of these tusks.
- c) Use your stem-and-leaf diagram to calculate the mean and median mass of the tusks. Explain how you used your stem-and-leaf diagram.
- d) Compare the mean mass of the tusks in the caché with the mass of an elephant tusk described in this excerpt taken from the Internet. Explain any discrepancy between these masses.
- e) Using the *Guinness Book of Records*, find and record the mass of the heaviest elephant tusk ever discovered. Why is this mass different from that described in the excerpt taken from the Internet?
- f) Explain how you would estimate the value of the ivory in the caché when ivory sells for \$50 per kilogram. State your estimate.
- ②. a) Use the information given in the article from *THE ECONOMIST* to estimate the mean mass of the tusks exported between 1979 and 1987.
- b) Calculate the mean mass of the tusks of the elephants killed in the Kenya-Tanzania border area, using the information in the *African Wildlife News* excerpt.
- c) Compare your answers in a) and b) and explain why they may differ.

THE IVORY WARS

This sample of articles shows how the market for ivory (called “white gold”) caused poachers to think of elephants only in terms of the ivory in their tusks.

THE ECONOMIST July 1989

Saving the Elephant

The African Elephant like the blue whale and the black rhino is on a fast track to extinction. The African elephant's misfortune is its tusks [and] their main cash value as sources of ivory. ... The group at Imperial College reckon that 6828 tonnes of ivory were exported between 1979 and 1987. ... That suggests that between 680 000 and 760 000 elephants must have been killed during that period.

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African Wildlife News May - June 1997
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THE IVORY WARS

- ①. The conservation authorities discovered a hidden caché of 40 elephant tusks. These tusks probably came from the slaughter of a small herd. They recorded the mass of each tusk (shown below) to the nearest kilogram.

14	22	16	9	8	13	21	6
36	17	20	5	35	36	22	16
31	16	23	13	3	12	8	10
20	35	14	22	3	36	37	7
8	10	31	3	17	11	8	3

- Display the masses given above in a stem-and-leaf diagram. Would a newspaper use a stem-and-leaf to report data? Explain why or why not.
- How would you use the stem-and-leaf to find the masses of the heaviest and lightest tusks? Suggest reasons why there is such a difference in the sizes of these tusks.
- Use your stem-and-leaf diagram to calculate the mean and median mass of the tusks. Explain how you used your stem-and-leaf diagram.
- Compare the mean mass of the tusks in the caché with the mass of an elephant tusk described in this excerpt taken from the Internet. Explain any discrepancy between these masses.
- Using the *Guinness Book of Records*, find and record the mass of the heaviest elephant tusk ever measured. Why is this mass different from that described in the excerpt taken from the Internet?
- Explain how you would estimate the value of the ivory in the caché when ivory sells for \$50 per kilogram. State your estimate.

Loxodonta Africana
Giant African elephants, whose tusks alone can weigh more than 45 kg each, are remarkable not only for their size but for ...

<http://encarta.msn.com/schoolhouse/environ/envelep2.htm>

- ②.
 - Use the information given in the article from THE ECONOMIST to estimate the mean mass of the tusks exported between 1979 and 1987.
 - Calculate the mean mass of the tusks of the elephants killed in the Kenya-Tanzania border area, using the information in the *African Wildlife News* excerpt.
 - Compare your answers in a) and b) and explain why they may differ.

GRADE 7

ANSWER KEY FOR ACTIVITY 3

1. a) Stem-and-Leaf Diagram for Masses of the Tusks

tens	ones
0	3 3 3 3 5 6 7 8 8 8 8 9
1	0 0 1 2 3 3 4 4 6 6 6 7 7
2	0 0 1 2 2 2 3
3	1 1 5 5 6 6 6 7

- b) heaviest tusk: 37 kg
lightest tusk: 3 kg

Responses will vary; e.g. “The lightest tusk is probably from an immature elephant, and the heaviest tusk from an adult male.”

- c) The median mass is the mean of the two middle values, 14 and 16; i.e. median mass is 15 kg.

The mean mass is the sum of all the masses divided by 40; i.e. $677 \div 40 = 16.925$ kg.

- d) The mean mass of the tusks in the caché is 16.925 kg, while the Internet excerpt indicates that an elephant tusk “can weigh more than 45 kg”. This phrase suggests that 45 kg is not the average, but an upper limit.

- e) The Internet excerpt indicates that an elephant tusk “can weigh more than 45 kg”. This phrase suggests that 45 kg is not the average, but near an upper limit. However, the Guinness Book of Records gives the mass of the heaviest tusk ever found to be 108 kg. This is the perhaps the most massive tusk of all time and therefore a rare extreme.

- f) The total mass of all the tusks in the caché is 677 kg. At a price of \$50 per kilogram, the caché would sell for:

$$677 \times \$50 = \$33\,850.$$

2. a) The ECONOMIST article indicates that 6828 tonnes of ivory were exported between 1979 and 1987. This is a total mass of 6 828 000 kg.

If 680 000 elephants yield 6 828 000 kg of ivory, then the mean mass of ivory per elephant is:

$$6\,828\,000 \text{ kg} \div 680\,000 \approx 10 \text{ kg}.$$

If 760 000 elephants yield 6 828 000 kg of ivory, then the mean mass of ivory per elephant is:

$$6\,828\,000 \text{ kg} \div 760\,000 \approx 9 \text{ kg}.$$

These estimates assume that the mean mass of ivory yielded by an elephant is between 9 and 10 kg. Since an elephant has two tusks, the estimates assume that the mean mass of ivory in a tusk is between 4.5 and 5 kg.

- b) The *African Wildlife News* estimates that it took 1000 elephants to yield 5 tonnes of ivory. That is, 1000 elephants yield 5000 kg of ivory, so on average, an elephant yields 5 kg of ivory. This suggests that each tusk has a mass of about 2.5 kg.

- c) The elephants in the ECONOMIST article were killed between 1979 and 1987, while those in the *African Wildlife News*, were killed around 1996. Possibly, as the elephant populations dwindled, poachers began to slaughter herds of elephants, including females and immature males rather than just the mature males. This would cause a reduction in the mean tusk mass. It could also be argued that the elephants in the Kenya-Tanzania border were atypical of elephants in other regions.

The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 3				
	Level 1	Level 2	Level 3	Level 4
APPLICATION				
Constructed Stem-and-Leaf Diagram (exercise ❶ a)) DM 7-3	<ul style="list-style-type: none"> •Stem-and-leaf diagram seriously flawed: poorly labelled, and/or incorrectly constructed. 	<ul style="list-style-type: none"> •Stem-and-leaf diagram has correct structure, but there are minor errors or omissions. 	<ul style="list-style-type: none"> •Stem-and-leaf diagram is correctly constructed. 	<ul style="list-style-type: none"> •In addition, the stem-and-leaf diagram is clearly labelled and presented.
APPLICATION				
Mean, Median and Extreme Values obtained from Stem-and-Leaf Diagram (exercises ❶ b) & c)) DM 7-7, 7-9	<ul style="list-style-type: none"> •Incorrect calculations. Explanations missing. 	<ul style="list-style-type: none"> •Some correct calculations. •Some explanations given. 	<ul style="list-style-type: none"> •Mostly correct calculations. •Reasonable explanations given. 	<ul style="list-style-type: none"> •Correct calculations. •Clear explanations given.
PROBLEM SOLVING				
Problem Solving involving the Concept of the Mean (exercises ❶ d), e), f) and ❷) DM 7-13, 7-15	<ul style="list-style-type: none"> •Answers contain several errors and/or omissions. •Explanations missing. 	<ul style="list-style-type: none"> •Answers are mostly correct, but contain some errors. •Some explanations given. 	<ul style="list-style-type: none"> •Answers are mostly correct. •Reasonable explanations are given. 	<ul style="list-style-type: none"> •Almost all answers are correct, and the explanations are complete and clearly expressed.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: CONSTRUCTED STEM-AND-LEAF PLOTS

Level 1

Level 2

This diagram is seriously flawed. The student has not realized that the leaves must be recorded as often as they occur.

Level 3

This diagram is constructed correctly but the leaves have not been arranged in order. Consequently the student has recorded incorrect values for the median and extreme.

Level 4

This diagram has been constructed correctly and applied correctly, but it has no labels.

This diagram is constructed correctly and labelled clearly.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: MEAN, MEDIAN, & EXTREME VALUES

Level 2

Level 4

Two entries, and 8 and a 20 are missing from the stem-and-leaf plot. This results in an incorrect median value. Explanations are missing.

All calculations are correct. (The mean was rounded up to 17.) Clear explanations are given.

PROBLEM SOLVING INVOLVING THE CONCEPT OF THE MEAN

Level 4

Level 4

Almost all the answers were correct. There was an error in the calculation of the mean mass of the tusks exported between 1979 and 1987. Also the student forgot to divide by 2 to get the mass of one tusk. Explanations were complete and clearly expressed.

Almost all the answers were correct. There was an error in the calculation of the value of ivory. Also the student forgot to divide by 2 to get the mass of one tusk. The student cleverly averaged the estimates of the number of elephants. Explanations were complete and clearly expressed.

ACTIVITY 4 – TEACHER EDITION

THE ANIMAL VS. HUMAN COMPETITION FOR FOOD

Expectations Addressed

- DM 7-1 • demonstrate the pervasive use of data
- DM 7-2 • understand the impact that statistical methods have on decision making.
- DM 7-6 • search data bases for information and interpret the numerical data.
- DM 7-14 • display data on bar graphs, pictographs, and circle graphs, with and without the help of technology
- DM 7-15 • make inferences that are based on data analysis
- DM 7-16 • evaluate arguments that are based on data analysis.

Context

By the time they have finished Activity 1, most students are typically outraged by the ivory trade, while still unaware of the full dimensions of the problem. In this activity, we present some real articles, taken from the Internet, that present the problem as it is seen by some of the Africans. Some claim that they depend upon the ivory trade for their survival. Others, the farmers, complain that the elephants trample their crops and destroy their livelihood. Still others argue that the ivory trade is an important vehicle for raising money to provide reserves for the elephant. Canadians may recognize that this problem has many of the elements of our concerns about the depleting cod resources in the Grand Banks. It is important in our role as educators that we avoid the temptation to support a particular position, but instead introduce the students to both sides of the argument and encourage them to contemplate the quandry and make their own decisions.

In their report, students are to indicate the extent to which the ban on the ivory trade has been successful. The only evidence provided in the activities was the observation that the survival status of the elephant was changed from endangered to threatened. While this is one measure they might include in their report, it is hoped that the Internet investigations will acquaint them with other recent measures that have been undertaken to preserve the African elephant. On June 21, 1997, an article in the *ECONOMIST* suggested that there has been some recent success in allowing a restricted trade in ivory by countries that have an abundance of elephants. New estimates of the elephant populations and more recent reports on this situation will emerge as months pass and may change the class perspectives on this imminent crisis. Students should surf the Internet periodically to discover whether there is any change in the status of the elephant, the ivory trade, or the problem of the vanishing habitat for elephants. They can report their findings to the class.

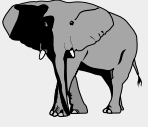
ACTIVITY 4 – STUDENT PAGE

ELEPHANT VS. HUMAN COMPETITION FOR FOOD

Not everyone wants to see the elephant population increase. The screen display below was accessed from this Washington Post website.

washingtonpost.com | home page | site index | search

Can Elephants and Humans Coexist in Africa?
 By Lynne Duke
 Post Foreign Service
 Thursday, June 19, 1997.



But who will protect Mukuba?
 "If the elephant finishes my crops, to whom can I go for assistance?", he asked in his native Siyeyi language. ... In this Sangwali District village west of Zimbabwe's Victoria Falls, crop-poor farmers are reduced to beggary when herds of elephants ravage their crops and destroy their livelihood.

...and the following appeared on the website of the Australian Conservation Gazette.

<http://www.infoweb.com.au/conservation/art11.htm>

Ivory Trade: Who Decides?

Is it fair for ivory trade to be looked at from a purely conservation perspective when elephants are a prime resource in countries such as Zimbabwe where poverty is rife? Should countries outside of Africa decide what's best for countries whose elephant populations are , in many cases, at nuisance levels and responsible for widespread crop destruction?
 ... Kofi Osei, Chairperson of the African Community Organization, is passionate about these issues and believes that western interference is depriving African nations the opportunity to develop a lucrative and sustainable industry [in ivory trade].

ACTIVITY 4 – TEACHER EDITION

The Lesson Launch 5 minutes

In this lesson, we want to have students conduct research and gather data and information from the Internet. In the process, they will discover the other side of the debate about the ivory trade. We might initiate the discussion as follows:

In the previous activities, we learned that poachers in search of ivory were slaughtering herds of elephants and contributing to the threat of its extinction. Although no one wants the elephant to become extinct, there are many in Africa who do not want a ban on the ivory trade. There are others who argue that the ivory trade is good for elephants. Who, besides the poacher, would want this, and what possible arguments could they make?

Rather than discussing possible answers at this point, ask the students to think about this question as they surf the Internet sites that will be given on their activity sheets.

Cooperative Learning Activity 45minutes

Organize the students in groups of three. Appoint a chair of each group, who is to assign one member the job of recorder, and the other, the job of reporter. The chair is responsible for having the group work through exercises ❶ through ❺. The recorder is responsible for recording their answers in a single report. After the group has completed this report, it will be the reporter's responsibility to read the group's answer to exercise ❺ including reasons why the group believes the elephant will or will not become extinct in their lifetime.

Ensure that some books or periodicals are available for students to glean for information while the other students are working at the computer. Encourage the students to gather data where possible, and to support their position with graphs. If several groups are not finished when 45 minutes have elapsed, allow them to finish the work at home. Students with access to the Internet at home often prefer to work there and will invest substantial time and energy browsing for elephant-related data.

Closure

When all groups have finished preparing their reports, have each reporter present the group's answer to question ❺. This is the opportune time to invite discussion and to encourage evaluation of various arguments. When a student finds a particularly appropriate website, record the URL on the blackboard so that others may visit that site and gather more information. When the discussion is concluded, students should be made aware of the concept of "sustainable use"; that is, to preserve elephants, whales, tigers, and other endangered species, we might consider harvesting a small number of them to raise money for their preservation. No one knows whether this is an effective approach, but it is one that is under serious consideration by some conservation groups.

ACTIVITY 4 – STUDENT PAGE

ELEPHANT VS. HUMAN COMPETITION FOR FOOD

RESEARCH REPORT

In this activity, research the following questions using references such as those given below. Then write a research report on your findings to answer these questions.

- ❶ Why is the elephant threatened with extinction?
- ❷ What measures have been taken to preserve the African elephant population?
- ❸ To what extent have these measures been effective?
- ❹ Why do some people want a ban on the sale of ivory, and why do others oppose such a ban?
- ❺ Do you think the African elephant will become extinct in your lifetime? Support your argument with reasons. Use graphs where you think appropriate.

Some sources you might investigate are given below.

Books & Periodicals

Chadwick, D. H. *The Fate of the Elephant*. Sierra Club Books, San Francisco. 492 pp.

DiSilvestro, R. L. (1991) *Among the Elephants*. Viking Press, New York. 285 pp.

Freeman, M. R. and U. P. Kreuter (Editors) 1994 *Elephants and Whales: Resources for Whom?* Gordon and Breach Science Publishers. 321 pp.

Orenstein, R. (1991) *Elephants. The Deciding Decade*. Sierra Club Books, San Francisco, CA 160 pp.

Internet Sites

http://www.wcmc.org.uk:80/species/data/species_sheets/afrielep.htm

<http://www.washingtonpost.com/wp-srv/frompost/features/june97/elephant.htm>

<http://wildnetafrica.com/wildlifearticles/refjournal/1996/database.html>

<http://www.si.edu/resource/faq/nmnh/elephant.htm>

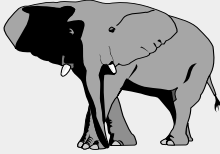
ELEPHANT VS. HUMAN COMPETITION FOR FOOD

Not everyone wants to see the elephant population increase. The screen display below was accessed from this Washington Post website.

[washingtonpost.com](http://www.washingtonpost.com) | [home page](#) | [site index](#) | [search](#)

Can Elephants and Humans Coexist in Africa?

By Lynne Duke
Post Foreign Service
Thursday, June 19, 1997.



But who will protect Mukuba?
“If the elephant finishes my crops, to whom can I go for assistance?”, he asked in his native Siyeyi language. ... In this Sangwali District village west of Zimbabwe’s Victoria Falls, crop-poor farmers are reduced to beggary when herds of elephants ravage their crops and destroy their livelihood.

...and the following appeared on the website of the Australian Conservation Gazette.

<http://www.infoweb.com.au/conservation/art11.htm>

Ivory Trade: Who Decides?

Is it fair for ivory trade to be looked at from a purely conservation perspective when elephants are a prime resource in countries such as Zimbabwe where poverty is rife? Should countries outside of Africa decide what’s best for countries whose elephant populations are , in many cases, at nuisance levels and responsible for widespread crop destruction?
... Kofi Osei, Chairperson of the African Community Organization, is passionate about these issues and believes that western interference is depriving African nations the opportunity to develop a lucrative and sustainable industry [in ivory trade].

ELEPHANT VS. HUMAN COMPETITION FOR FOOD

RESEARCH REPORT

In this activity, research the following questions using references such as those given below. Then write a research report on your findings to answer these questions.

- ❶** Why is the elephant threatened with extinction?
- ❷** What measures have been taken to preserve the African elephant population?
- ❸** To what extent have these measures been effective?
- ❹** Why do some people want a ban on the sale of ivory, and why do others oppose such a ban?
- ❺** Do you think the African elephant will become extinct in your lifetime? Support your argument with reasons. Use graphs where you think appropriate.

Some sources you might investigate are given below.

Books & Periodicals

Chadwick, D. H. *The Fate of the Elephant*. Sierra Club Books, San Francisco. 492 pp.

DiSilvestro, R. L. (1991) *Among the Elephants*. Viking Press, New York. 285 pp.

Freeman, M. R. and U. P. Kreuter (Editors) 1994 *Elephants and Whales: Resources for Whom?* Gordon and Breach Science Publishers. 321 pp.

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Internet Sites

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<http://wildnetafrica.com/wildlifearticles/refjournal/1996/database.html>

<http://www.si.edu/resource/faq/nmnh/elephant.htm>

GRADE 7

ANSWER KEY FOR ACTIVITY 4

Answers may vary significantly. However, highest marks should be awarded to reports that include new information beyond that given in the activities. It is expected that students will glean information from print and media resources to provide complete answers.

- ❶. This answer could be taken directly from p.24.

Three reasons why the elephant populations are threatened with extinction are:

- They have been killed in great numbers for their ivory tusks.
- The fast-growing African human population needs more land to grow food and is moving into the elephant territory and destroying their habitat.
- The Sahara desert is expanding into areas that once grew trees on which elephants could feed, but now those trees are gone.

- ❷. The ban on the international sale of ivory is one measure that was taken to reduce poaching and the slaughter of elephants. Another measure has been the creation of elephant preserves that have been set aside to enable elephant herds to flourish without the intrusion of human population. Attempts to irrigate the deserts and reclaim areas lost to the Sahara desert have also been aimed at stemming the drop in elephant populations. Money has been raised by organizations like the *World Wildlife Federation* to provide funds directed toward the preservation of elephants. Money has also been raised by creating game parks where elephants can be observed.

- ❸. From Activity 1, most students will be aware that the elephant survival status was downgraded from “endangered” to “threatened” in 1990. This suggests that the ban on ivory sales was somewhat effective. However, subsequent information has suggested that in areas such as Southern Africa, where the elephants have been carefully managed as a valued resource, there has been a renewal of the elephant populations. Furthermore some limited trade in ivory has generated sufficient revenue to fund various projects that feed and protect the elephants

- ❹. Some people want a ban on the sale of ivory to prevent the slaughter of elephants for their tusks. They argue their position on the basis of humanitarian grounds, or out of concern that this marvellous animal may soon become extinct.

Others who profit from the sale of ivory argue that removal of the ivory trade would interfere with their livelihood. Farmers whose crops are trampled by marauding elephants also argue that their livelihoods are jeopardized by the elephant.

- ❺. There is no right or wrong answer to this question. The important thing is that the student state a position and articulate a well-formulated response to support that position. Level 4 performance requires that the student understand the arguments on both sides of the ivory ban, and that the student can use the information gleaned from research to marshall a carefully reasoned response to the original question, “Will the African elephant become extinct in your lifetime?”. A premium should be placed on answers that provide graphs or other quantitative information to support their arguments.

The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 4				
	Level 1	Level 2	Level 3	Level 4
COMMUNICATION				
Information gleaned from electronic or print resources DM 7-6	•Report shows little or no evidence of information beyond that presented in the activities.	•Report shows some evidence of information obtained from resources beyond the activities.	•Report shows significant evidence of research beyond the activities.	•In addition the report shows the creative selection of new information to support the position taken.
COMMUNICATION				
Reporting of required knowledge related to concepts, procedures, & problem solving DM 7-15, 7-16	•Report appears disorganized and many responses are unclear.	•Report addresses most of the questions, but some responses are unclear.	•Report is well organized, and addresses all of the questions.	•In addition, the report contains clear articulation of the ideas involved and offers persuasive support for the position taken.

During the field testing of this unit in 1998, many students commented that they liked the activities. It was evident in their reports that they enjoyed doing research on the Internet, and that they had become deeply interested in the plight of the African elephant. Some were outraged by the slaughter of elephants for their ivory, while others balanced this concern with an awareness of the struggle for survival of the African people. Throughout the process, most students were deeply engaged in the learning process.

WHAT YOU MIGHT SEE

REPORTING OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 1

This report addresses some of the questions, but several of the responses are difficult to interpret. The student sees the threat posed by the trade in ivory, but does not give any evidence of an understanding of the reason why some people oppose the ban on ivory, except that the tusks are “really good looking”. There is however, some evidence that the student gathered information from sources other than that presented in the activities.

WHAT YOU MIGHT SEE

REPORTING OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 2

This report addresses most of the questions. Several of the responses are too brief and not sufficiently comprehensive, “On the other hand despicable people want ivory because its luxurious and its sort of expensive”. The issue of the expanding populations and the loss of habitat to this cause is mentioned but not discussed. However the student does attempt to refer to the graphs to support the position that elephants will die out if we don’t take measures.

WHAT YOU MIGHT SEE

REPORTING OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 3

This report addresses all the questions. It identifies the people who oppose the ban on ivory because they want ivory for jewelry, piano keys, etc., but does not discuss the plight of those whose livelihood depends on ivory or farming. The opinion that the African elephant will not become extinct during her lifetime is not supported with a persuasive argument of any kind.

WHAT YOU MIGHT SEE

REPORTING OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 4

This report addresses all the questions. It is well organized into paragraphs that deal with each issue in turn. The writing is articulate and the ideas flow in a logical sequence. The student has identified all the major issues raised in the activities and there is evidence of research beyond the information provided in those activities. The opinion that the elephant will not become extinct is supported by reference to the many conserving practices that the student has described in the second section of the report.



PART IV

PROBABILITY

in

Grade 8

THE ONTARIO CURRICULUM, GRADES 1-8: MATHEMATICS

DATA MANAGEMENT AND PROBABILITY: GRADE 8

Overall Expectations

By the end of Grade 8, students will:

- systematically collect, organize, and analyse primary data;
- use computer applications to examine and interpret data in a variety of ways;
- interpret displays of data and present the information using mathematical terms;
- evaluate data and draw conclusions from the analysis of data;
- identify probability situations and apply a knowledge of probability;
- appreciate the power of using a probability model by comparing experimental results with theoretical results.

Specific Expectations

(For convenient reference, the specific expectations are coded. DM 8-1 refers to the first data management expectation in grade 8. P 8-3 denotes the third probability expectation in grade 8.)

Students will:

Collecting and Organizing Data

- DM 8-1** - collect primary data using both a whole population (census) and a sample of classmates, organize the data on tally charts and stem-and-leaf plots, and display the data on frequency tables;
- DM 8-2** - understand the relationship between a census and a sample;
- DM 8-3** - read a database or spreadsheet and identify its structure;
- DM 8-4** - manipulate and present data using spreadsheets, and use the quantitative data to solve problems;
- DM 8-5** - search databases for information and use the quantitative data to solve problems;

Analysing Data

- DM 8-6** - know that a pattern on a graph may indicate a trend;
- DM 8-7** - understand and apply the concept of the best measure of central tendency;
- DM 8-8** - discuss trends in graphs to clarify understanding and draw conclusions about the data;
- DM 8-9** - discuss the quantitative information presented on tally charts, stem-and-leaf plots, frequency tables, and/or graphs;

Analysing Data (cont'd)

- DM 8-10 - explain the choice of intervals used in constructing bar graphs or the choice of symbols in pictographs;
- DM 8-11 - assess bias in data-collection methods;
- DM 8-12 - read and report information about data presented on line graphs, comparative bar graphs, pictographs, and circle graphs, and use the information to solve problems;
- DM 8-13 - determine the effect on a measure of central tendency of adding or removing a value (e.g., what happens to the mean when you add or delete a very low or very high data entry);

Concluding and Reporting

- DM 8-14 - understand the difference between a bar graph and a histogram;
- DM 8-15 - construct line graphs, comparative bar graphs, circle graphs, and histograms, with and without the help of technology, and use the information to solve problems (e.g., extrapolate from a line graph to predict a future trend);
- DM 8-16 - make inferences and convincing arguments that are based on data analysis;
- DM 8-17 - evaluate arguments that are based on data analysis;
- DM 8-18 - determine trends and patterns by making inferences from graphs;
- DM 8-19 - explore with technology to find the best presentation of data;

Probability

- P 8-1 - use probability to describe everyday events;
- P 8-2 - identify 0 to 1 as a range from "never happens" (impossibility) to "always happens" (certainty) when investigating probability;
- P 8-3 - list the possible outcomes of simple experiments by using tree diagrams, modelling, and lists;
- P 8-4 - identify the favourable outcomes among the total number of possible outcomes and state the associated probability (e.g., of getting chosen in a random draw);
- P 8-5 - use definitions of probability to calculate complex probabilities from tree diagrams and lists (e.g., for tossing a coin and rolling a die at the same time);
- P 8-6 - compare predicted and experimental results;
- P 8-7 - apply a knowledge of probability in sports and games, weather predictions, and political polling.

ACTIVITY 1 – TEACHER EDITION

IS THE WORLD SERIES RIGGED?

Expectations Addressed

- P 8-3 • list the possible outcomes of simple experiments by using tree diagrams, modelling, and lists.
- P 8-4 • identify the favourable outcomes among the total number of possible outcomes and state the associated probability.
- P 8-7 • apply a knowledge of probability in sports and games, weather predictions, and political polling.

Context

Every year, the top team in the *American League* plays the top team in the *National League*, to determine the best major league baseball team in North America. In this contest, the two teams play until one team has won a total of four games. That team is declared the *World Series Champion*. Since the *World Series* is a “best four out of seven” contest, the duration of that series is somewhere between four and seven games. However, when the *World Series* lasts seven games, people sometimes speculate that it is “rigged”, so that the team owners can make more money from the additional games. This conjecture often promotes debate and discussion, but few people are sufficiently adept in probability theory to test this hypothesis.

This unit challenges students with the question, “Is the *World Series* rigged?”. In Activity 1, the student is introduced to the context of the problem, and guided through a review of the fundamental language of probability. Such terms as *frequency*, *relative frequency*, *outcomes*, *equally likely*, and *simulate* are reviewed in this context. Students are then introduced to the idea of using the outcomes in the toss of a fair coin to simulate the outcome of a game between two evenly matched teams. Activity 2 builds on this idea, and pairs the students in a “best four out of seven” coin toss simulation of a *World Series*. Students toss a coin until either heads or tails has occurred a total of 4 times, and they record the number of tosses required. (This is called a “trial”) They repeat this simulation for 20 trials, tallying the number of trials that required 4, 5, 6, and 7 tosses. By calculating the relative frequency of each of these possible outcomes, students obtain an estimate of the true probability of each outcome. In Activity 3, students actually calculate the probability of each outcome using tree diagrams. Activity 4 ties the previous investigations together, by having students compare the theoretical probability found in Activity 3 with the experimental probability found in Activity 2, and then attempt to reconcile these probabilities with the observed occurrence of the 7-game series over the past 50 years.

ACTIVITY 1 – STUDENT PAGE

IS THE WORLD SERIES RIGGED?

Another World Series Ends in 7!

DO YOU THINK THE WORLD SERIES IS RIGGED TO MAKE IT LAST 7 GAMES? THAT WAY THEY GET MORE MONEY.

THE DATA SHOWS THAT THE WORLD SERIES WENT 7 GAMES 23 TIMES IN 50 YEARS.

WHAT IS THE PROBABILITY THAT THE WORLD SERIES WILL LAST 7 GAMES?

Number of Games in a World Series 1947 – 1997				
	4 games	5 games	6 games	7 games
Frequency	7	8	12	23

ACTIVITY 1 – TEACHER EDITION

The Lesson Launch 5 minutes

This lesson can be launched by asking a series of questions such as the following to ensure all students understand the context of the problem:

- *What is the World Series in Baseball?*
- *How many games in a World Series?*
- *What is the fewest number of games possible in a World Series? What is the greatest number of games possible?*

Once it is clear that students understand that a World Series is a “best four out of seven” series, and that winning requires that a team win a *total* of four games (not four games in a row), proceed to discuss the terms, *outcome*, *equally likely outcome*, *frequency*, and *relative frequency*.

Initiating Activity 15 minutes

Provide students with the student pages of Activity 1, and assign them exercises ❶ → ❹. Students should work individually through these exercises and record their answers in their notebook. As you walk around the room, you will be able to determine which students are having difficulty with the terminology or the fundamental ideas. Allow students who finish ahead of the rest to proceed to exercise ❺.

Individual Activity 25 minutes

Discuss the answers to questions ❶ through ❹, to ensure the students are prepared for the simulation. Before students embark upon this activity, it is important that they understand how a tossing of a coin simulates a game between equally matched teams. Ask students whether we can use a fair coin to simulate a game between two teams that are not equally matched, and ask them to explain why not. Then ask them how we might simulate a game between two teams that were unevenly matched so that one team has twice the chance of winning of the other team. (Make a spinner divided into three equal parts with two parts shaded the colour of the team favoured to win.)

Instruct them to use a coin and follow the directions given in exercise ❺. As you circulate around the class, check that the students are tallying the outcomes to show how many trials terminate in 7 tosses. Check also that they are able to determine the relative frequency of the outcome “7 tosses”. Students should gradually discover what they might have deduced logically; that is, the probability that a World Series will go 6 games is the same as the probability it will last 7 games.

Closure

When all the students have finished, invite each group to write the relative frequency of outcome “7 tosses” on the blackboard. Then invite the students to make a conjecture about which outcome is more likely – “6 tosses” or “7 tosses”. Ask them how many times out of 100 they would expect a series, with one team at 3 wins and the other at 2 wins, to go exactly 6 games. Then ask them how many times it would go 7 games. At this point, the light may go on for many students.

ACTIVITY 1 – STUDENT PAGE

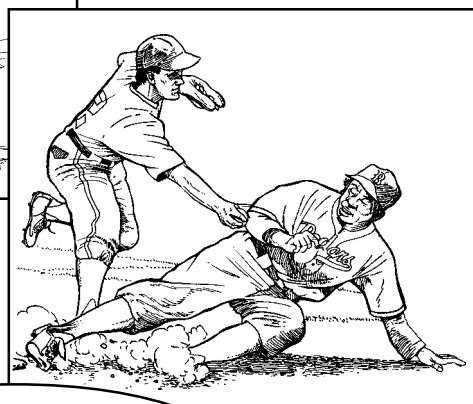
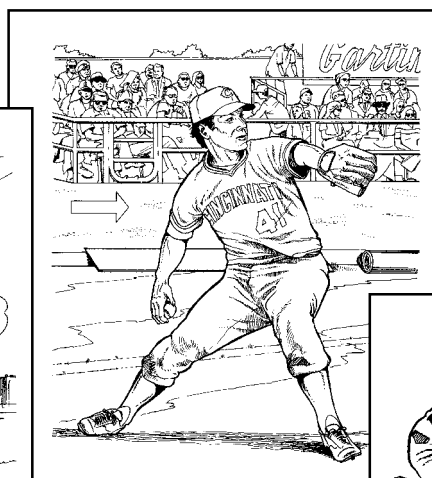
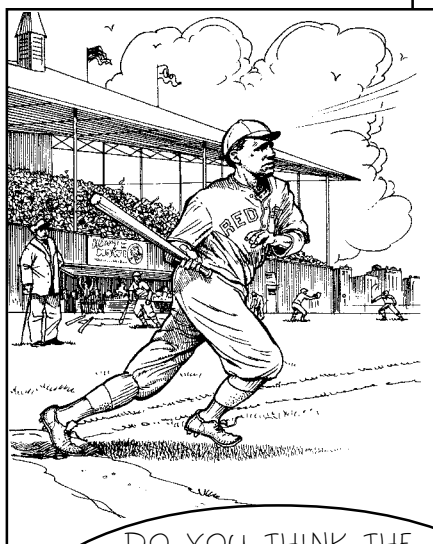
- ❶. A hockey game has three possible outcomes: win, lose, or tie.
 - a) What are the possible outcomes of a World Series baseball game?
 - b) Do you think these outcomes are equally likely? Explain.
- ❷. In the 1997 season, the Blue Jays had 76 wins and 86 losses.
 - a) What was the total number of games played?
 - b) What was the frequency of wins?
 - c) What was the relative frequency of wins?
 - d) What was the relative frequency of losses?
 - e) What is the sum of the relative frequencies in c) and d)? Why?
- ❸. The World Series is a “best 4 out of 7 series”. This means that two teams play until one team has won 4 games. That team is declared the winner.

One possible outcome of a World Series is that there are 7 games.

 - a) List all the other possible outcomes.
 - b) Do you think all the outcomes you listed are equally likely? Explain.
 - c) Which outcome do you think is most likely? Give reasons.
- ❹. The Blue Jays defeated the Atlanta Braves in 6 games in the 1992 World Series.
 - a) How many games had the Atlanta Braves and the Blue Jays each won after the fifth game? (List all the possibilities.)
 - b) After 5 games, the Blue Jays and the Braves were given an equal chance of winning the sixth game. At that point was it more likely that the World Series would last 6 games or that it would last 7 games. Give reasons for your answer.
- ❺. If the Blue Jays and the Braves are evenly matched, we can “simulate” a game by tossing a coin. If it comes up heads (H), we call it a Blue Jay win, and if tails (T), we call it a Blue Jay loss. Suppose after the fifth game, the Blue Jays have won 3 games and lost 2.
 - a) Toss a coin to determine whether the World Series ends in the sixth game or goes to a seventh game.
 - b) Repeat part a) 20 times and record how many times the World Series goes 7 games.
 - c) Use your tally in part b) to decide whether it is more likely that a World Series will go 6 games or 7 games.
 - d) Compare your answer to part c) with the answers of other students. What do you conclude?

IS THE WORLD SERIES RIGGED?

Another World Series Ends in 7!



DO YOU THINK THE WORLD SERIES IS RIGGED TO MAKE IT LAST 7 GAMES? THAT WAY THEY GET MORE MONEY.

THE DATA SHOWS THAT THE WORLD SERIES WENT 7 GAMES 23 TIMES IN 50 YEARS.



WHAT IS THE PROBABILITY THAT THE WORLD SERIES WILL LAST 7 GAMES?

Number of Games in a World Series 1947 – 1997				
	4 games	5 games	6 games	7 games
Frequency	7	8	12	23



ACTIVITY 1 – STUDENT ACTIVITY PAGE

IS THE WORLD SERIES RIGGED?

- ❶. A hockey game has three possible outcomes: win, lose, or tie.
 - a) What are the possible outcomes of a World Series baseball game?
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- ❷. In the 1997 season, the Blue Jays had 76 wins and 86 losses.
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 - e) What is the sum of the relative frequencies in c) and d)? Why?
- ❸. The World Series is a “best 4 out of 7 series”. This means that two teams play until one team has won 4 games. That team is declared the winner. One possible outcome of a World Series is that there are 7 games.
 - a) List all the other possible outcomes.
 - b) Do you think all the outcomes you listed are equally likely? Explain.
 - c) Which outcome do you think is most likely? Give reasons.
- ❹. The Blue Jays defeated the Atlanta Braves in 6 games in the 1992 World Series.
 - a) How many games had the Atlanta Braves and the Blue Jays each won after the fifth game? (List all the possibilities.)
 - b) After 5 games, the Blue Jays and the Braves were given an equal chance of winning the sixth game. At that point was it more likely that the World Series would last 6 games or that it would last 7 games. Give reasons for your answer.
- ❺. If the Blue Jays and the Braves are evenly matched, we can “simulate” a game by tossing a coin. If it comes up heads (H), we call it a Blue Jay win, and if tails (T), we call it a Blue Jay loss. Suppose after the fifth game, the Blue Jays have won 3 games and lost 2.
 - a) Toss a coin to determine whether the World Series ends in the sixth game or goes to a seventh game.
 - b) Repeat part a) 20 times and record how many times the World Series goes 7 games.
 - c) Use your tally in part b) to decide whether it is more likely that a World Series will go 6 games or 7 games.
 - d) Compare your answer to part c) with the answers of other students. What do you conclude?

GRADE 8
ANSWER KEY FOR ACTIVITY 1

1. a) Two possible outcomes: win or lose (If the teams are tied after 9 innings, the game goes into overtime until a team wins.)
b) A tie is usually less likely than a win or a loss. Also, unless two teams are evenly matched, there is usually one team which is favoured to win, so win and lose are not equally likely outcomes for a particular team.
2. a) Total games: $76 + 86 = 162$
b) Frequency of wins: 76
c) Relative frequency of wins: $76/162 \approx 0.47$
d) Relative frequency of losses: $86/162 \approx 0.53$
e) The sum is exactly 1. The frequency of wins plus frequency of losses is equal to the total number of games, so dividing the total frequencies by the total number of games yields 1.
3. a) 4 games, 5 games, or 6 games.
b) If one team is much better than the other, then 4 games is the most likely outcome. However, if the teams are evenly matched, 4 games is less likely than a larger number of games.
c) Since both teams have to be very good to get to the World Series, they will usually be fairly evenly matched, so it is more likely that the World Series will last 6 or 7 games.
4. a) Since the Blue Jays won the World Series in 6 games, they must have won the sixth game and that must have been their fourth win. Therefore after 5 games they must have had 3 wins, leaving Atlanta with 2 wins.
b) After 5 games, the Blue Jays had 3 wins and the Braves had 2 wins. If the Blue Jays win the sixth game, that gives them four wins, so the series ends in 6 games. If the Braves win the sixth game, the series is tied 3 wins each, so the World Series must go to the seventh game. If both teams are given an equal chance of winning the sixth game, then a 6-game World Series has the same probability as a 7-game World Series.
5. a) Answers will vary.
b) After 20 tosses, the frequency of World Series that go 7 games should be somewhere between 7 and 13.
c) See Part b)
d) On comparison of their answers to Part c) with other students, they will discover that their classmates come up with different answers. This suggests the two outcomes are equally likely and that all the variation is due to chance.

The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 1				
	Level 1	Level 2	Level 3	Level 4
CONCEPTS				
Understanding of the key concepts of Probability Simulation (exercises ① - ③) P 8-3, 8-7	<ul style="list-style-type: none"> •Significant difficulty identifying outcomes as equally likely or not. •Serious errors in calculating frequencies and relative frequencies. 	<ul style="list-style-type: none"> •Some difficulty determining whether outcomes are equally likely or not. •Explanation of why relative frequencies total 1 is unclear. 	<ul style="list-style-type: none"> •Frequencies and relative frequencies calculated correctly •Identifies correctly outcomes that are and are not equally likely. 	<ul style="list-style-type: none"> •In addition to Level 3, states conditions under which outcomes are equally likely. •Provides reasons for answers given
PROBLEM SOLVING				
Problem Solving (exercises ④ - ⑤)	<ul style="list-style-type: none"> •Unable to determine that the Braves won 2 and Blue Jays 3 after 5 games, and to deduce that the probabilities of 6-game and 7-game World Series are equal. 	<ul style="list-style-type: none"> •Determines that the Braves won 2 and Blue Jays 3 after 5 games, but unable to deduce that the probabilities of 6-game and 7-game World Series are equal. 	<ul style="list-style-type: none"> •Is able to deduce that the probabilities of 6-game and 7-game World Series are equal. 	<ul style="list-style-type: none"> •In addition to Level 3, attributes differences in observed and predicted frequencies of 6 and 7-game series to chance variation.

ACHIEVEMENT LEVELS DEFINED BY THE MINISTRY OF EDUCATION AND TRAINING

- Level 1** Identifies achievement that falls much below the provincial standard.
- Level 2** Identifies achievement that approaches the standard.
- Level 3** The “provincial standard”, identifies a high level of achievement of provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.
- Level 4** Identifies achievement that surpasses the standard.

WHAT YOU MIGHT SEE

UNDERSTANDING THE KEY CONCEPTS OF PROBABILITY SIMULATION

Level 2

The student has expressed relative frequency of wins as wins per loss instead of wins per game. No evidence that the student is able to judge relative likelihood of outcomes of a best 4 out of 7.

Level 3

Frequencies and relative frequencies are calculated correctly. Good explanation of relative likelihood of outcomes of a best 4 out of 7, but list of outcomes contains some non-permissible outcomes such as 5, 2.

WHAT YOU MIGHT SEE

UNDERSTANDING THE KEY CONCEPTS OF PROBABILITY SIMULATION

Level 4

The student retyped the questions and wrote answers in italics. Answers show clear understanding of frequencies, relative frequencies. Although the student misinterpreted question 1b), he was able to explain in 3c) that if two teams are equally matched, then the win/lose outcomes are equally likely.

PROBLEM SOLVING

Level 2

Level 3

The answers to 4b) and 5c) suggest that the student has not understood how to relate the likelihood of a 7-game series to the likelihood of a 6-game series.

The student understands how to relate the likelihood of a 7-game series to the likelihood of a 6-game series. To achieve at Level 4, the student would have to realize that the simulation results might be a chance variation.

ACTIVITY 2 – TEACHER EDITION

SIMULATING A WORLD SERIES

Expectations Addressed

- DM 8-1** • collect primary data, organize the data on tally charts, and display the data on frequency tables.
- DM 8-9** • discuss the quantitative information presented on tally charts, frequency tables, and/or graphs.
- DM 8-15** • construct line graphs, comparative bar graphs, circle graphs, and histograms, with and without the help of technology, and use the information to solve problems.
- DM 8-18** • determine trends and patterns by making inferences from graphs.
- P 8-7** • apply a knowledge of probability in sports and games, weather predictions, and political polling.

Context

This activity builds upon exercise 5 in Activity 1 in which students were introduced to the concept of a simulation. By simulating the outcome of a game between two evenly matched teams with the toss of a coin, the student is able to compute the relative frequencies of each of the four possible outcomes i.e. 4 games, 5 games, 6 games, and 7 games, in a best 4 out of 7 series. The relative frequency for any outcome can be expected to be a close approximation to the true *theoretical probability* when there is a large number of trials. When the number of trials is large, the relative frequency of an outcome is sometimes referred to as the *experimental probability*.

The purpose of exercises 3 through 6 is to help students discover that there is a great deal of variation in the experimental probabilities of the four outcomes from group to group when the simulation is run for only 20 trials. However, by combining the data of all the groups into class data, we have effectively increased the number of trials to several hundred and can expect to obtain better estimates of the probability of each outcome. This is an important principle in statistics, that should be emphasized with the students. That is, as we increase sample size, we can expect to increase the accuracy of our estimates.

In exercise 8, students represent the probabilities of the four outcomes in a circle graph. This emphasizes the fact that the sum of the probabilities is 1. Exercise 9, is provided for students who have access to a graphing calculator. Program BLUEJAYS is provided in the answer key for those with a TI-73 or TI-83 graphing calculator.

ACTIVITY 2 – STUDENT PAGE

SIMULATING A WORLD SERIES

In Exercise 5 of Activity 1, you learned that we can simulate the outcomes of a World Series between two evenly matched teams by tossing a coin. In this activity, you will toss a coin to estimate the probabilities that a World Series will last 4, 5, 6 or 7 games.

You will need a partner and a coin to do this activity. Choose one of you to be the coin tosser and the other to be the recorder.

- The coin tosser flips the coin until either 4 heads or 4 tails are obtained. The recorder tallies the number of heads and tails like this.

HEADS	TAILS
/ / / /	\\

← It took 6 tosses to get 4 heads or tails.



This is called *one trial* of the simulation. The recorder tallies the results of this trial in a table like this.

FREQUENCIES OF OUTCOMES

4 TOSSES	5 TOSSES	6 TOSSES	7 TOSSES
		/	

← The first trial required 6 tosses.

- Run 20 trials and tally your results in the **FREQUENCY OF OUTCOMES** table you created in 1.

- Your teacher will display this Class **FREQUENCY OF OUTCOMES** table on the overhead projector or blackboard.

	4 Tosses	5 Tosses	6 Tosses	7 Tosses	Total
group 1	2	8	7	3	20
group 2	3	9	4	4	20
.					
.					
Class Total					

Go to the overhead projector or blackboard and record your frequencies in this class frequency table.

- Calculate the total frequency of each outcome in the class frequency table, and the total number of trials. (Each group had 20 trials.) Record your totals in a table like this.

FREQUENCIES OF OUTCOMES FOR THE CLASS

4 TOSSES	5 TOSSES	6 TOSSES	7 TOSSES
30	71	103	96

ACTIVITY 2 – TEACHER EDITION

The Lesson Launch 5 minutes

This lesson should build upon the simulation that students conducted as they worked through exercise 5 of Activity 1. Pose the question:

Is it more likely that a World Series between two evenly matched teams will end in 6 games or in 7 games?

Ask for a show of hands to indicate those who believe 6 games is more likely. Repeat for 7 games. Stimulate debate by asking students to provide reasons for their position. If a student suggests that the two outcomes are equally likely (the correct answer), ask how we might check whether this is true. We could combine all the data collected in exercise 5 of Activity 1 and compare the relative frequencies of both outcomes. This might suggest equal likelihood is feasible but, “How can we be sure?”. You can help students deduce that the probabilities are equal by starting at the point where one team has won 3 games and the other team has won 2 and asking, “What is the probability that the series will end in 6 games?” Once students realize this is the same question as “What is the probability that the team with 3 wins will win the next game?”, they have the answer.

Paired Activity 30 minutes

Have the students choose partners. Each pair should have one person who is the coin tosser and the other who maintains a tally of the outcomes. Ensure that students know how to prepare a “frequency of outcomes” chart and to tally frequencies. Assign exercises 1 and 2 of Activity 2 to the class. Most students will have completed 20 tosses in 5 minutes.

Display your class Frequency of Outcomes table on the overhead projector, and have each group enter their frequencies on the transparency as shown.

	4 Tosses	5 Tosses	6 Tosses	7 Tosses	Total
group 1	2	8	7	3	20
group 2	3	9	4	4	20
•					
•					
•					
Class Total					

Have each group work through exercises 4, 5, 6, 7, and 8 using the data from the class table. If there is sufficient access, encourage students to use the computer to construct their graphs. This gives them additional exposure to spreadsheets and the results are neater. If students have graphing calculators, download BLUEJAYS from your calculator using the link cable and encourage them to play with that simulation.

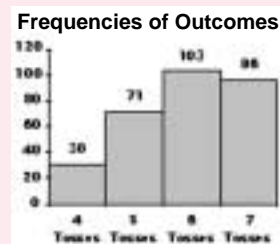
Closure

Discuss with the students the results of the class simulation, and ask them to ensure that they have recorded the relative frequencies of each outcome in their notebooks, because they will need this information later. If you have a graphing calculator with an overhead projector display, run BLUEJAYS for 100, 200 and 1000 trials and compare the results of that simulation with the class simulation.

ACTIVITY 2 – STUDENT PAGE

SIMULATING A WORLD SERIES

5. Make a histogram like this to show the frequency of each outcome given in your table in 4.



6. Study your histogram. In a best 4 out of 7 tosses:
- Which outcome do you think is more likely, 6 tosses or 7 tosses?
 - Which of the four possible outcomes do you think is the most likely?

7. Use the table in 4 to calculate the relative frequency of each of the four outcomes in a best 4 out of 7 toss. Record your answers in a table like this.

Outcome	Relative Frequency
4 Tosses	$30/300 = 0.1$
5 Tosses	$71/300 = 0.24$
6 Tosses	$103/300 = 0.34$
7 Tosses	$96/300 = 0.32$

8. If you have access to a word processor, make a circle graph showing the relative frequencies of the outcomes as shown in your table in 7. Print out the circle graph and tape it into your notebook.

9. If you have a graphing calculator, download program BLUEJAYS. Program BLUEJAYS simulates a best 4 out of 7 coin toss. On each trial, heads(H) stands for a Blue Jay win, and tails(T) stands for a Blue Jay loss. This is the same as simulating the number of games in a World Series between two evenly matched teams.

- Run BLUEJAYS for 20 trials.
- Record the frequency of 7 games in a World Series. Calculate the relative frequency of 7 games.
- Compare the relative frequency in Part b) with the relative frequency of 7 tosses in your table in 7.
- Run BLUEJAYS for 100 trials and 500 trials and compare the relative frequency of each outcome with the relative frequency given in 7.
- Does increasing the number of trials yield relative frequencies closer to the values in 7? Explain.



ACTIVITY 2 – STUDENT ACTIVITY PAGE

SIMULATING A WORLD SERIES

In exercise 5 of Activity 1, you learned that we can simulate the outcomes of a World Series between two evenly matched teams by tossing a coin. In this activity, you will toss a coin to estimate the probabilities that a World Series will last 4, 5, 6 or 7 games.

You will need a partner and a coin to do this activity. Choose one of you to be the coin tosser and the other to be the recorder.

- The coin tosser flips the coin until either 4 heads or 4 tails are obtained. The recorder tallies the number of heads and tails like this.

HEADS	TAILS
	\ \

← It took 6 tosses to get 4 heads or tails.



This is called *one trial* of the simulation. The recorder tallies the results of this trial in a table like this.

FREQUENCIES OF OUTCOMES			
4 TOSSES	5 TOSSES	6 TOSSES	7 TOSSES

← The first trial required 6 tosses.

- Run 20 trials and tally your results in the **FREQUENCY OF OUTCOMES** table you created in 1.

- Your teacher will display this Class **FREQUENCY OF OUTCOMES** table on the overhead projector or blackboard.

Go to the overhead projector or blackboard and record your frequencies in this class frequency table.

	4 Tosses	5 Tosses	6 Tosses	7 Tosses	Total
group 1	2	8	7	3	20
group 2	3	9	4	4	20
.					
.					
.					
Class Total					

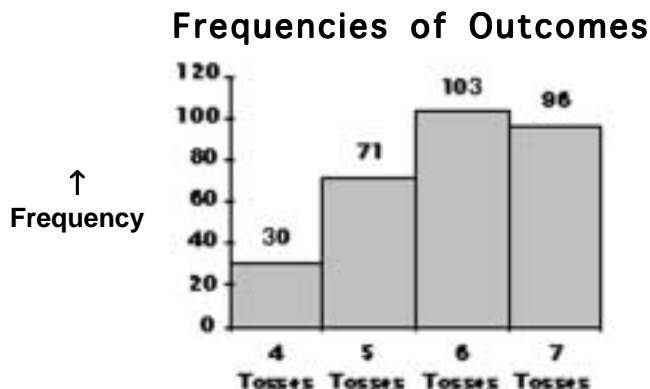
- Calculate the total frequency of each outcome in the class frequency table, and the total number of trials. (Each group had 20 trials.) Record your totals in a table like this.

FREQUENCIES OF OUTCOMES FOR THE CLASS			
4 TOSSES	5 TOSSES	6 TOSSES	7 TOSSES
30	71	103	96

ACTIVITY 2 – STUDENT ACTIVITY PAGE

SIMULATING A WORLD SERIES

5. Make a histogram like this to show the frequency of each outcome given in your table in 4.



6. Study your histogram. In a best 4 out of 7 tosses:
- Which outcome do you think is more likely, 6 tosses or 7 tosses?
 - Which of the four possible outcomes do you think is the most likely?

7. Use the table in 4 to calculate the relative frequency of each of the four outcomes in a best 4 out of 7 toss. Record your answers in a table like this.

Outcome	Relative Frequency
4 Tosses	$30/300 = 0.1$
5 Tosses	$71/300 = 0.24$
6 Tosses	$103/300 = 0.34$
7 Tosses	$96/300 = 0.32$

8. If you have access to a word processor, make a circle graph showing the relative frequencies of the outcomes as shown in your table in 7. Print out the circle graph and tape it into your notebook.
9. If you have a graphing calculator, download program BLUEJAYS. Program BLUEJAYS simulates a best 4 out of 7 cointoss. On each trial, *heads*(H) stands for a Blue Jay win, and *tails* (T) stands for a Blue Jay loss. This is the same as simulating the number of games in a World Series between two evenly matched teams.
- Run BLUEJAYS for 20 trials.
 - Record the frequency of 7 games in a World Series. Calculate the relative frequency of 7 games.
 - Compare the relative frequency in Part b) with the relative frequency of 7 tosses in your table in 7.
 - Run BLUEJAYS for 100 trials and 500 trials and compare the relative frequency of each outcome with the relative frequency given in 7.
 - Does increasing the number of trials yield relative frequencies closer to the values in 7? Explain.



GRADE 8 **ANSWER KEY FOR ACTIVITY 2**

① and ②. Answers will vary.

Students' **FREQUENCY OF OUTCOMES** table should show tally marks that total 20.

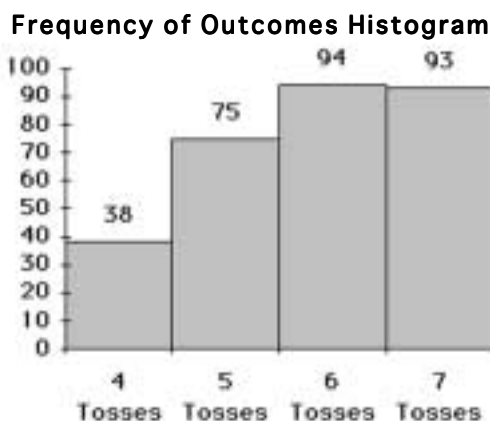
FREQUENCIES OF OUTCOMES

4 TOSSES	5 TOSSES	6 TOSSES	7 TOSSES
///	////	//// //	////

③. A class of 30 students will have 15 pairs and therefore, a total of 300 trials. The class frequency table will have frequencies in roughly the proportions shown here.

	4 Tosses	5 Tosses	6 Tosses	7 Tosses	Total
group 1	2	8	7	3	20
group 2	3	9	4	4	20
•					
•					
•					
Class Total	38	75	94	93	300

④ and ⑤. Answers will vary from class to class, but with the large number of trials conducted by the entire class, the histogram can be expected to look something like this.

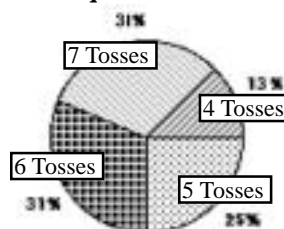


⑥. a) Both outcomes are equally likely. The two frequencies should be very close: some students will infer that the 6-toss and 7-toss outcomes are equally likely, while others will take the data verbatim and conclude that the outcome with the higher frequency is more likely.

b) The theoretical probabilities of the outcomes are respectively $1/8$, $1/4$, $5/16$ and $5/16$ for 4, 5, 6, and 7 tosses. Students should infer that 4 and 5 tosses are less likely than 6 and 7 tosses.

⑦ and ⑧. The relative frequencies of 4, 5, 6, and 7 tosses should be close to the true probabilities 0.125, 0.25, 0.3125, and 0.3125 respectively. This yields a circle graph like the one below.

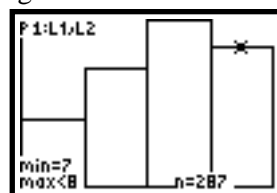
Relative Frequencies of Outcomes



⑨. a) Program BLUEJAYS for the TI-73 and TI-83 graphing calculators is shown in the display.

b) & c) For 20 trials, there may be a significant difference between the relative frequencies.

d) When we ran BLUEJAYS for 1000 trials, and pressed TRACE, we obtained this histogram, showing the relative frequency of 7 games to be 0.287.



```

PROGRAM:BLUEJAYS
:Input "HOW MANY TRIALS",N
:(4,5,6,7)→L1
:4→dim(L2)
:0→L2
:1→Xscl:4→Xmin
:8→Xmax:0→Ymin
:For(I,1,N)
:0→A
:For(J,1,7)
:randInt(0,1)→X
:A+X→A
:If max(A,J-A)=4
:Then
:Disp "NUMB OF GAMES"
:Disp J
:L2(J-3)+1→L2(J-3)
:7→J
:End
:End
:End
:PlotsOn 1
:Plot1(Histogram
:L1,L2)
:max(L2)+1→Ymax
:DispGraph
:
  
```


The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 2				
	Level 1	Level 2	Level 3	Level 4
APPLICATION				
Constructed Histogram (exercise ⑤) DM 8-14, 8-15	<ul style="list-style-type: none"> •Graph incomplete and poorly labelled. •No evidence that student understands what is to be plotted. 	<ul style="list-style-type: none"> •Histogram seriously flawed: poorly labelled, and/or incorrect frequencies, i.e. do not match frequencies in the class frequency table. 	<ul style="list-style-type: none"> •Histogram mainly correct but some minor labelling or presentation errors. 	<ul style="list-style-type: none"> •Histogram correct and appropriately labelled.
APPLICATION				
Constructed Circle Graph (exercises ⑦ and ⑧) DM 8-15	<ul style="list-style-type: none"> •Circle graph seriously flawed or missing. 	<ul style="list-style-type: none"> •Circle graph shows incorrect relative frequencies and has minor labelling errors or deficiencies. 	<ul style="list-style-type: none"> •Circle graph has correct relative frequencies and minor labelling errors or deficiencies. 	<ul style="list-style-type: none"> •Circle graph is correctly labelled and well presented.

ACHIEVEMENT LEVELS DEFINED BY THE MINISTRY OF EDUCATION AND TRAINING

- Level 1** Identifies achievement that falls much below the provincial standard.
- Level 2** Identifies achievement that approaches the standard.
- Level 3** The “provincial standard”, identifies a high level of achievement of provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.
- Level 4** Identifies achievement that surpasses the standard.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: CONSTRUCTED HISTOGRAM

Level 2

The histogram has no title. Neither of the axes is labelled and there are spaces between the bars. There are no numbers on the bars, so it is not possible to determine if the frequencies total a multiple of 20.

Level 3

Level 4

The histogram is mainly correct, but there is no title, no label on the vertical axis, and there are spaces between the bars. The frequencies are shown on the bars and their sum is a multiple of 20, suggesting that the student probably recorded them correctly.

The histogram is correct and appropriately labelled. It would be useful to have the vertical axis labelled “frequency”, but the title of the graph enables us to determine what is graphed.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: CONSTRUCTED CIRCLE GRAPH

Level 1

Level 3

The relative frequencies have been calculated incorrectly. The frequencies in the table do not total 280 and the student has computed the *reciprocals* of the relative frequencies. The graph is incorrectly titled “Frequencies” instead of “Relative Frequencies” of outcomes.

The circle graph has the correct relative frequencies and is appropriately labelled. There are no percentages or decimal numbers given on the sectors of the circle to enable the reader to read the actual relative frequencies.

Level 4

The circle graph is correctly labelled and well presented.

ACTIVITY 3 – TEACHER EDITION

USING TREE DIAGRAMS TO CALCULATE PROBABILITIES

Expectations Addressed

- P 8-3** • list the possible outcomes of simple experiments by using tree diagrams, modelling, and lists.
- P 8-4** • identify the favourable outcomes among the total number of possible outcomes and state the associated probability.
- P 8-5** • use definitions of probability to calculate complex probabilities from tree diagrams and lists (e.g. for tossing a coin and rolling a die at the same time).
- P 8-7** • apply a knowledge of probability in sports and games, weather predictions, and political polling.

Context

This activity develops the concept of the probability of an event as the fraction of all possible *equally likely* outcomes that are favourable to that event. For example, in the toss of two coins, there are four equally likely possible outcomes, denoted by HH, HT, TH, and TT. Then the probability of the event, “not both heads or both tails” is the number of ways that event can occur (i.e. HT or TH) divided by the number of equally likely outcomes, i.e. $2 \div 4$ or $1/2$. We write:

$$\text{Probability (not HH or TT)} = \frac{\overset{\text{favourable outcomes}}{\downarrow} \#\{\text{HT, TH}\}}{\underset{\text{total number of outcomes}}{\uparrow} \#\{\text{HH, HT, TH, TT}\}} = \frac{1}{2}$$

where # denotes the number of outcomes in the brackets.

Exercises ① and ② introduce the student to the representation of the outcomes of the toss of three coins, by the notation

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

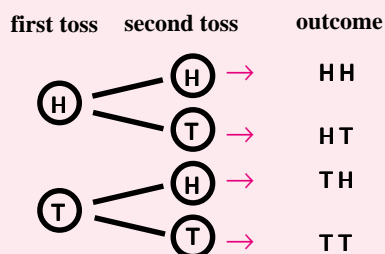
The student learns how to construct all possible outcomes using a tree diagram and then to calculate the probability of any event by dividing the number of outcomes favourable to that event by the total number of outcomes.

Exercises ③ through ⑤ help students construct progressively more complex tree diagrams corresponding to the toss of 3 coins and building to the toss of 5 coins. Using these tree diagrams, they calculate in exercise ⑥ the probabilities of each outcome in a best 3 out of 5 coin toss. In exercises w through y the students apply reasoning similar to that in Activity 1, to calculate the probability of 7 games in a best 4 out of 7 series. This sets the stage for the comparison of experimental and theoretical probability broached in Activity 4.

ACTIVITY 3 – STUDENT PAGE

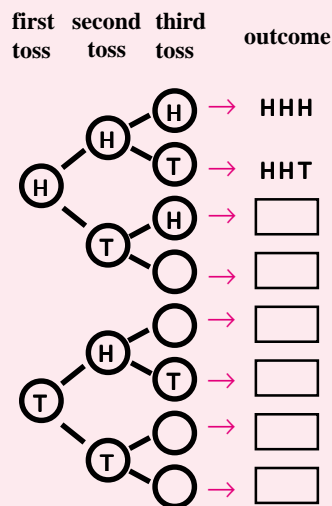
USING TREE DIAGRAMS TO CALCULATE PROBABILITIES

When we toss a coin, there are two possible outcomes: *heads* (H) and *tails* (T). When we toss a coin twice, there are four possible outcomes, as shown in the *tree diagram* below.



- ①. Explain the meaning of the outcome HT. Is this the same as the outcome TH? Explain.

- ②. Suppose we toss a coin 3 times. There are eight possible outcomes. Copy and complete this tree diagram to show all eight outcomes.



- ③. Suppose that two people play best 2 out of three in a coin toss. That is, the winner is the first person to win two tosses. Suppose also that one player always chooses heads and the other tails.
- List the outcomes in your tree diagram for which there is no winner after the first two tosses.
 - What is the probability that there is no winner after the first two tosses?
 - What is the probability that three tosses are needed to determine a winner?
- ④. Make a tree diagram like the one in ② above, but expand it to show all possible outcomes when 4 coins are tossed. How many outcomes are there? List all possible outcomes, and check that they are all different.

ACTIVITY 3 – TEACHER EDITION

The Lesson Launch 5 minutes

This lesson should begin with a discussion of the limitations of simulations.

Is it possible that a simulation with a large number of trials could yield an incorrect probability? Why?

Through discussion, students should discover that simulations can give very accurate estimates of true probabilities, particularly as the number of trials is increased. However, the results of a simulation are not as reliable as an actual calculation of a (theoretical) probability. The importance of simulations is two fold:

- simulations can serve as a check on our computation of the theoretical probability of an event
- simulations can be used to estimate probabilities of events for which no theoretical probability can be computed.(e.g. the probability that a player will win in a game of solitaire.)

Ask students how we might calculate the probability that a best 2 out of 3 coin toss will last 3 games. Invite students who think they know how to share their ideas with the class.

Individual Activity 30 minutes

Distribute the student pages for Activity 3. Using the overhead projector or blackboard, present the tree diagram for the toss of two coins. Ask students to give the probabilities of events such as HT, “not HT”, and “either HT or TH”. Remind students how we divide favourable outcomes by the total number of outcomes, and emphasize that this is valid only if all outcomes are equally likely.

Have the students do exercises ①, ②, and ③ in their notebooks. Allow students who finish early to proceed with the later exercises. Within about 10 minutes, take up these exercises with the class to ensure that everyone knows how to create a tree diagram and calculate the probability that a “best 2 out of 3 tosses” will require 3 games.

Hand out the template for the tree diagram corresponding to the toss of a coin five times. Have each student work through exercises ④ through ⑨. During this activity, it is important to circulate around the classroom to help students who have difficulty completing the tree diagrams, or who do not understand how to identify the outcomes that are favourable to a particular event. A quick way to check that a student has generated all the outcomes in a coin toss experiment, is to count the number of outcomes in the student’s list. In the toss of n coins, there should be 2^n outcomes.

Closure

Discuss with the students the probabilities they obtained for the outcomes associated with the toss of 5 coins, 6 coins and 7 coins. Discuss the limitations of tree diagrams when the number of coins is large. Finally, ensure that students make the connection between the outcomes in a “best 4 out of 7” coin toss and the possible number of games in a World Series.

ACTIVITY 3 – STUDENT PAGE

USING TREE DIAGRAMS TO CALCULATE PROBABILITIES

- a) Complete the tree diagram on your template to show all possible outcomes when a coin is tossed 5 times. Record the outcomes in a list.
b) How many outcomes are there? Check that all your outcomes are different.
c) How many outcomes do you think there would be if you tossed a coin 6 times?
- Suppose two people play best 3 out of 5 in a coin toss.
a) Put a ✓ beside each outcome in your list in ⑤ that shows no winner after the first 4 tosses.
b) Write the probability that there is no winner after the first 4 tosses.
c) Write the probability that 5 tosses are needed to determine a winner.
- Record all the outcomes from your list in ⑤ in which:
a) heads occurs 3 times and tails twice.
b) heads occurs twice and tails 3 times.
- a) A coin is tossed 5 times and the outcome is HTHHT. The coin is then tossed one more time and it turns up tails. Write the outcome for the 6 tosses of the coin.
b) A coin is tossed 5 times and the outcome is one of those in your list in ⑦. The coin is then tossed one more time. Write all possible outcomes for the 6 tosses of the coin.
c) How many of the outcomes in your list in Part b) show 3 heads and 3 tails?
d) What is the probability that 6 tosses of a coin will yield 3 heads and 3 tails?
- Suppose two people play the best 5 out of 7 in a coin toss.
a) How many possible outcomes are there on the toss of 7 coins?
b) On how many of these outcomes is there no winner after the first 6 tosses?
c) What is the probability that 7 tosses are needed to determine a winner?

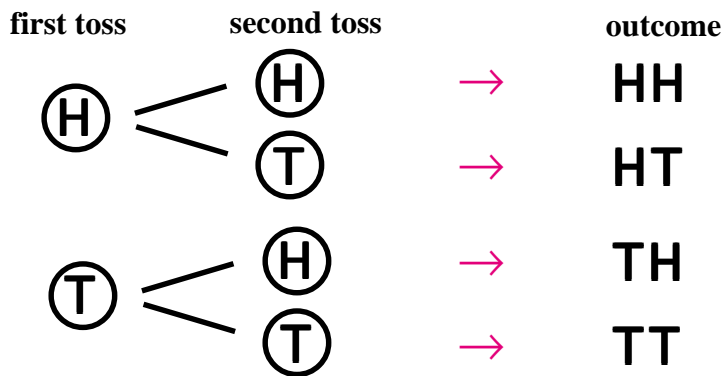
CHALLENGE

Use your tree diagram to determine the probability of each outcome of a best 4 out of 7 series. Using a computer, construct a spinner that will simulate such a series on each spin.

ACTIVITY 3 – STUDENT ACTIVITY PAGE

USING TREE DIAGRAMS TO CALCULATE PROBABILITIES

When we toss a coin, there are two possible outcomes: *heads* (H) and *tails* (T). When we toss a coin twice, there are four possible outcomes, as shown in the *tree diagram* below.



- ①. Explain the meaning of the outcome HT. Is this the same as the outcome TH? Explain.

- ②. Suppose we toss a coin 3 times. There are eight possible outcomes. Copy and complete this tree diagram to show all eight outcomes.

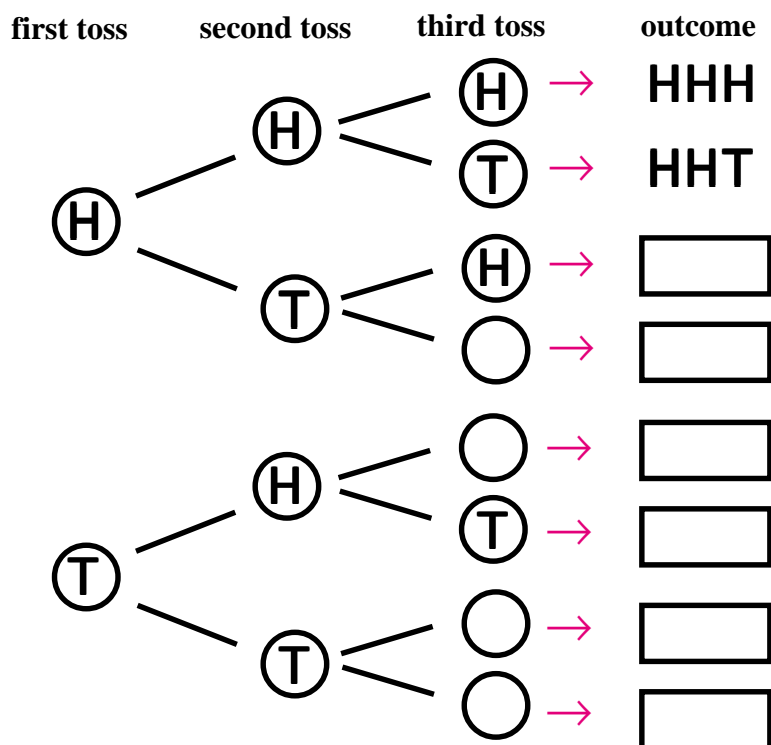
- ③. Suppose that two people play best 2 out of three in a coin toss. That is, the winner is the first person to win two tosses. Suppose also that one player always chooses heads and the other tails.

a) List the outcomes in your tree diagram for which there is no winner after the first two tosses.

b) What is the probability that there is no winner after the first two tosses?

c) What is the probability that three tosses are needed to determine a winner?

- ④. Make a tree diagram like the one in ② above, but expand it to show all possible outcomes when 4 coins are tossed. How many outcomes are there? List all possible outcomes, and check that they are all different.



ACTIVITY 3 – STUDENT ACTIVITY PAGE

USING TREE DIAGRAMS TO CALCULATE PROBABILITIES

5. a) Complete the tree diagram on your template to show all possible outcomes when a coin is tossed 5 times. Record the outcomes in a list.
b) How many outcomes are there? Check that all your outcomes are different.
c) How many outcomes do you think there would be if you tossed a coin 6 times?
6. Suppose two people play best 3 out of 5 in a coin toss.
a) Put a ✓ beside each outcome in your list in 5 that shows no winner after the first 4 tosses.
b) Write the probability that there is no winner after the first 4 tosses.
c) Write the probability that 5 tosses are needed to determine a winner.
7. Record all the outcomes from your list in 5 in which:
a) heads occurs 3 times and tails twice.
b) heads occurs twice and tails 3 times.
8. a) A coin is tossed 5 times and the outcome is HTHHT. The coin is then tossed one more time and it turns up tails. Write the outcome for the 6 tosses of the coin.
b) A coin is tossed 5 times and the outcome is one of those in your list in 7. The coin is then tossed one more time. Write all possible outcomes for the 6 tosses of the coin.
c) How many of the outcomes in your list in Part b) show 3 heads and 3 tails?
d) What is the probability that 6 tosses of a coin will yield 3 heads and 3 tails?
9. Suppose two people play the best 5 out of 7 in a coin toss.
a) How many possible outcomes are there on the toss of 7 coins?
b) On how many of these outcomes is there no winner after the first 6 tosses?
c) What is the probability that 7 tosses are needed to determine a winner?

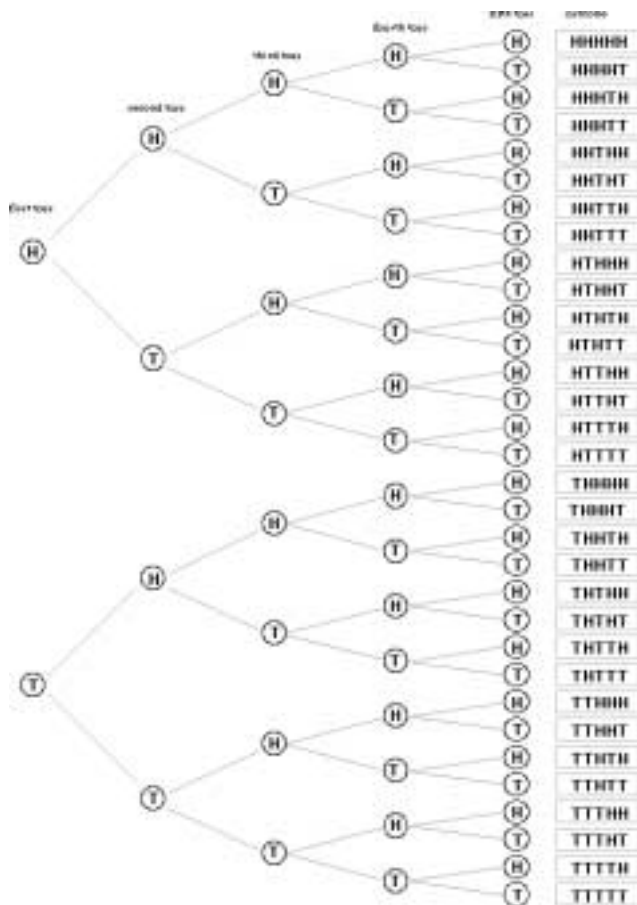
CHALLENGE

Use your tree diagram to determine the probability of each outcome of a best 4 out of 7 series. Using a computer, construct a spinner that will simulate such a series on each spin.

GRADE 8

ANSWER KEY FOR ACTIVITY 3

- ① HT represents the outcome in which a head occurs on the first toss and a tails on the second toss, but TH represents the outcome in which a tail occurs first, followed by a head. Therefore they represent different outcomes and are not the same.
- ③ a) The outcomes for which there is no winner in a best 2 out of 3 are those which do not begin with HH or TT; i.e., HTH, HTT, THH, and THT.
b) The four outcomes given in Part a) are half of the outcomes, so the probability that there is no winner after two tosses is 0.5.
c) If there is no winner after the first two tosses, then a third toss is needed. Therefore the probability that a third toss is needed is 0.5.
- ⑤ a) The completed template is shown below.
b) There are 2^5 or 32 outcomes.
c) There would be 2^6 or 64 possible outcomes.
- ②. The eight outcomes listed from the top of the tree diagram to the bottom are:
HHH, HHT, HTH, HTT, THH, THT, TTH, TTT
- ④. There are 16 possible outcomes when four coins are tossed. We can generate eight of them by placing an H in front of each of the outcomes listed in exercise ② above, to obtain:
HHHH, HHHT, HHHT, HHTT, HTHH, HTHT, HTTH, HTTT
Then we generate the other eight by placing a T in front of each of the outcomes listed in exercise ② above, to obtain:
THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT
- ⑥. a) There is no winner after 4 tosses only if there are exactly 2 heads and 2 tails in the first four tosses. There are 12 outcomes that satisfy this condition.
HHTTH, HHTTT, HTHTH, HTHTT, HTTHH, HTTHT, THTHH, THTHT, TTHHH, TTHHT, THTTH, THTTT



- b) Since there is no winner after four tosses on 12 of 32 equally likely outcomes, the probability that there is no winner after four tosses is $12/32$ or $3/8$.¹
- c) The probability that 5 tosses are needed is the same as the probability that there is no winner after four tosses, i.e., $3/8$ or 0.375.
- ⑦. a) & b) There are 10 such outcomes in both cases.
- ⑧. a) The outcome is HTHHTT.
b) If we choose outcome HHHTT, possible outcomes for 6 tosses are: HHHTTH and HHHTTT.
c) Of these two outcomes, only one has 3 heads and 3 tails.
d) The probability of 3 heads and 3 tails on 6 tosses of a coin is half the probability of 3 of one and 2 of the other on 5 tosses of a coin; i.e. half of $20/32$ which is half of $10/16$ or $5/16$.
- ⑨. a) The number of outcomes is 2^7 or 128.
b) From 8d), there is no winner in $5/16$ of the outcomes; i.e. on $5/16 \times 128$ or 40 outcomes.
c) From 9b) the probability is $5/16$.

¹We can use combinatorics to check this result for ourselves, by computing the number of 4-letter arrangements of two H's and 2 T's as $4!/4$ or 6 out of 16 possible 4-letter arrangements.)

The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 3				
	Level 1	Level 2	Level 3	Level 4
APPLICATION				
Computation of Probabilities from Tree Diagrams (exercises ❶ – ❷) P 8-3, P 8-4, P 8-5	•Significant difficulty constructing tree diagrams and using them to calculate probabilities.	•Some errors in constructing tree diagrams, and serious errors in using them to calculate probabilities.	•Tree diagrams constructed correctly, and most probabilities calculated correctly from them	•Tree diagrams constructed correctly and almost all probabilities calculated correctly from them.
PROBLEM SOLVING				
Problem Solving (exercises ❸ – ❹)	•Few of the exercises answered correctly.	•A significant number of the exercises answered incorrectly.	•Most of the exercises answered correctly.	•Almost all of the exercises answered correctly.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: PROBABILITIES FROM TREE DIAGRAMS

Level 1

This student is having difficulty constructing tree diagrams at any level. Following this field test, a template was provided (see p. 93) to help students generate a tree diagram. This should be used for the toss of 3 coins, 4 coins, and 5 coins until students see the pattern.

Level 2

The tree diagram for four tosses of a coin is missing a branch, but the correct set of outcomes is given. No tree diagram for five tosses is given. The probabilities given in the student's answer to exercises ⑥ and ⑦ are incorrect.

Level 3

The tree diagrams for four tosses and five tosses of a coin are constructed correctly. The probabilities in exercise ⑥ are correct, but a counting error led to slightly incorrect probabilities in exercise ⑦.

WHAT YOU MIGHT SEE

APPLICATION OF MATHEMATICAL PROCEDURES: PROBABILITIES FROM TREE DIAGRAMS

Level 4

The girl used a computer to construct this tree diagram for all the outcomes that begin with heads, and then for all the outcomes that begin with tails (shown above). Almost all the probabilities [a small slip in ⑥ c)] were calculated correctly from these diagrams.

PROBLEM SOLVING

Level 4



Hmmm...

Not only did this student answer almost all of the exercises correctly, but he celebrates his triumph in discovering that exercise 6c) is really exercise 6b) in disguise!

ACTIVITY 4 – TEACHER EDITION

PUTTING THE WORLD SERIES TO THE TEST

Expectations Addressed

- DM 8-5** • search databases for information and use the quantitative data to solve problems.
- DM 8-9** • discuss the quantitative information presented on tally charts, frequency tables, and/or graphs.
- DM 8-15** • construct line graphs, comparative bar graphs, circle graphs, and histograms, with and without the help of technology, and use the information to solve problems.
- DM 8-16** • make inferences and convincing arguments that are based on data analysis.
- DM 8-17** • evaluate arguments that are based on data analysis.
- DM 8-18** • determine trends and patterns by making inferences from graphs.
- P 8-1** • use probability to describe everyday events.
- P 8-4** • identify the favourable outcomes among the total number of possible outcomes and state the associated probability.
- P 8-6** • compare predicted and experimental results.
- P 8-7** • apply a knowledge of probability in sports and games, weather predictions, and political polling.

Context

The purpose of this activity is to culminate the work of the first three activities in a resolution of the initial question, *Is the World Series Rigged?*. In so doing, we have students compare their experimental probability that a “best 4 out of 7” contest will last 7 games, (obtained from their simulation in Activity 2) with the theoretical probability they computed in Activity 3.

Then they compare these probabilities with the actual relative frequency of the outcome “7 games” in the World Series over a period of 50 years. The actual relative frequency of 7 games is $23/50$ or 0.46 compared with a theoretical probability of $5/16$ or 0.31 . A formal statistical test of the hypothesis that the World Series is rigged reveals that such a discrepancy between the predicted proportion of 7-game World Series and actual proportion can be attributed to chance variation in about 10% of all cases, so there is not enough evidence to indicate that the World Series is rigged. Although such formal tests are well beyond students at this level, they will be able to observe the discrepancy between the observed and predicted results, and conjecture reasons why such a discrepancy has occurred.

ACTIVITY 4 – STUDENT PAGE

PUTTING THE WORLD SERIES TO THE TEST

1. Assume the Blue Jays play an equally matched team in the World Series. That is, assume that the two possible outcomes, *win* and *lose* are equally likely.
 - a) Explain how the outcome of this World Series can be simulated by a coin toss.
 - b) Suppose we use *heads* (H) to stand for a Blue Jays win, and *tails* (T) for a Blue Jays loss. How many games are indicated by the outcome HHTHTTH? How many Blue Jay wins are indicated?
2. In the World Series, the Blue Jays play until they either win 4 games or lose 4 games. To estimate the probability that the World Series will last 7 games we can toss a coin until we get either 4 heads or 4 tails. Use your completed table in exercise 7 of Activity 2 to estimate the probability that the World series will last 7 games.
3. Use your answer in exercise 9 of Activity 3 to estimate the probability that the World series will last 7 games.
4. Compare your estimated probabilities in exercises 2 and 3. Are these estimates very close? Explain why or why not.
5. The actual number of games in each World Series for the 50 series between 1947 and 1997 is shown in this table. (Note there was no world series in 1994 because of the baseball strike.)



Number of Games in a World Series 1947 – 1997				
	4 games	5 games	6 games	7 games
Frequency	7	8	12	23

Calculate the relative frequencies for each of the outcomes in the table, and record them in a table like the one shown below.

NUMBER OF GAMES IN THE WORLD SERIES 1947-1997				
	4 GAMES	5 GAMES	6 GAMES	7 GAMES
FREQUENCY	7	8	12	23
RELATIVE FREQUENCY				

ACTIVITY 4 – TEACHER EDITION

The Lesson Launch 5 minutes

In this lesson, we need to link the experimental and theoretical probabilities derived in the previous activities to the actual relative frequency of the 7-game World Series. A discussion tying together these ideas can be stimulated with a question such as the following:

In Activity 2, we used a simulation to estimate the probability that a best 4 out of 7 coin toss will require 7 tosses. In Activity 3, we calculated this probability using tree diagrams. Do you think either of these methods gives us a good way of estimating the probability that a World Series will go 7 games? Explain why or why not.

Through discussion, students should build an understanding of how the outcomes in a best 4 out of 7 coin toss can be used to model the outcomes regarding the number of games in a World Series. They must also be reminded that this model is based on the assumption that the teams are evenly matched.

Individual Activity 15 minutes

Distribute the student pages for Activity 4. Have students work individually on exercises ① through ⑤. Check that the students' answers to exercises ②, ③ and ⑤ are correct, because they will all need to have the same answers in the cooperative learning activity to follow.

Cooperative Learning Activity 25 minutes

Organize the students in groups of three. Appoint a chair of each group, who is to assign one member the job of recorder, and the other, the job of reporter. The chair is responsible for having the group work through exercises ⑥ and ⑦. The recorder is responsible for recording their answers in a single report. After the group has completed this report, it will be the reporter's responsibility to read the group's answer to exercise 7 b) and to described the group's reasoning in deciding whether or not the World Series is rigged. While the slower groups are finishing their report, allow the groups who have finished their reports to proceed with the Internet exploration on the student activity page.

Closure

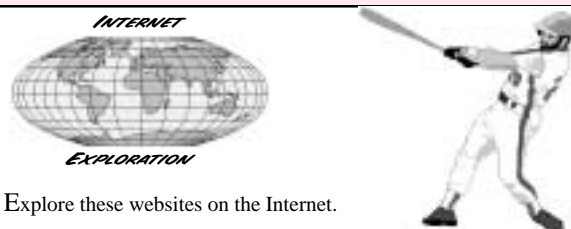
When all groups have finished preparing their reports, have each reporter present the group's answer to question 7b). This is the opportune time to invite discussion and to encourage evaluation of various arguments. Then discuss exercises ⑧ and ⑨ with the class. Develop with the class a list of the legitimate reasons why the World Series might go 7 games significantly more often than that predicted by probability theory. Such reasons might include the observation that teams may tend to let down when they have a two game lead, or the general tendency of teams to win their home games.

ACTIVITY 4 – STUDENT PAGE

PUTTING THE WORLD SERIES TO THE TEST

- ⑥. a) Use the table you created in Exercise ⑤ to make a circle graph showing the relative frequency of each number of games in a World Series.
b) Compare your circle graph with the one you constructed in Exercise ③ of Activity 2. How are they alike? different?
- ⑦. a) In Exercise ③, you estimated the probability that a World Series will last 7 games. Compare that estimate with the relative frequency of 7 games that you calculated in Exercise ⑤.

b) Do you think that the World Series is "rigged" to make it last 7 games?
Write a brief report to explain why or why not. Use the evidence you have gathered to support your argument.
- ⑧. a) Explain the meanings of the terms *relative frequency* and probability.
b) Write a few sentences to describe how we can use probability experiments to estimate probabilities. How can we ensure that the relative frequency is a good estimate of the probability?
- ⑨. a) When we use a coin toss experiment to simulate a game in a World Series, what assumption(s) are we making?
b) If the assumption(s) you identified in Part a) are not correct, will this tend to increase or decrease the probability that the World Series will last 7 games? Explain your answer.



Explore these websites on the Internet.

[http://www•majorleaguebaseball•com/library/postseas.sml](http://www.majorleaguebaseball.com/library/postseas.sml)
and
<http://www•majorleaguebaseball.com/library/alltime.sml>

What players had the highest batting averages for a season? Record the years when these averages were achieved. What does this information suggest about the skills of pitchers in the past and the present. Write a brief report on your conclusions.

ACTIVITY 4 – STUDENT ACTIVITY PAGE

PUTTING THE WORLD SERIES TO THE TEST

①. Assume the Blue Jays play an equally matched team in the World Series. That is, assume that the two possible outcomes, *win* and *lose* are equally likely.

- Explain how the outcome of this World Series can be simulated by a coin toss.
- Suppose we use *heads* (H) to stand for a Blue Jays win, and *tails* (T) for a Blue Jays loss. How many games are indicated by the outcome HHTHTTH? How many Blue Jay wins are indicated?



- In the World Series, the Blue Jays play until they either win 4 games or lose 4 games. To estimate the probability that the World Series will last 7 games we can toss a coin until we get either 4 heads or 4 tails. Use your completed table in exercise ⑦ of Activity 2 to estimate the probability that the World series will last 7 games.
- Use your answer in exercise ⑨ of Activity 3 to estimate the probability that the World series will last 7 games.
- Compare your estimated probabilities in exercises ② and ③. Are these estimates very close? Explain why or why not.
- The actual number of games in each World Series for the 50 series between 1947 and 1997 is shown in this table. (Note there was no world series in 1994 because of the baseball strike.)

Number of Games in a World Series 1947 – 1997				
	4 games	5 games	6 games	7 games
Frequency	7	8	12	23

Calculate the relative frequencies for each of the outcomes in the table, and record them in a table like the one shown below.

NUMBER OF GAMES IN THE WORLD SERIES 1947 – 1997				
	4 GAMES	5 GAMES	6 GAMES	7 GAMES
FREQUENCY	7	8	12	23
RELATIVE FREQUENCY				

ACTIVITY 4 – STUDENT ACTIVITY PAGE

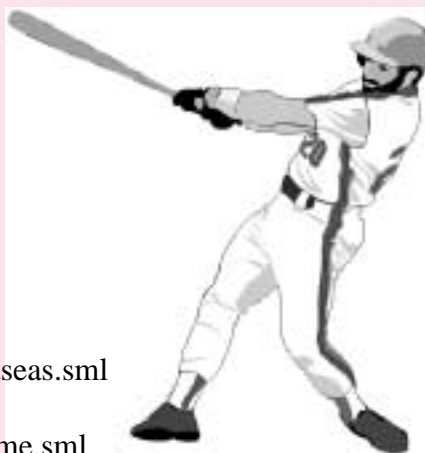
PUTTING THE WORLD SERIES TO THE TEST

6. a) Use the table you created in Exercise 5 to make a circle graph showing the relative frequency of each number of games in a World Series.
b) Compare your circle graph with the one you constructed in Exercise 8 of Activity 2. How are they alike? different?
7. a) In Exercise 3, you estimated the probability that a World Series will last 7 games. Compare that estimate with the relative frequency of 7 games that you calculated in Exercise 5.
b) Do you think that the World Series is “rigged” to make it last 7 games? Write a brief report to explain why or why not. Use the evidence you have gathered to support your argument.
8. a) Explain the meanings of the terms *relative frequency* and probability.
b) Write a few sentences to describe how we can use probability experiments to estimate probabilities. How can we ensure that the relative frequency is a good estimate of the probability?
9. a) When we use a coin toss experiment to simulate a game in a World Series, what assumption(s) are we making?
b) If the assumption(s) you identified in Part a) are not correct, will this tend to increase or decrease the probability that the World Series will last 7 games? Explain your answer.



Explore these websites on the Internet.

<http://www•majorleaguebaseball•com/library/postseas.sml>
and
<http://www•majorleaguebaseball.com/library/alltime.sml>



What players had the highest batting averages for a season? Record the years when these averages were achieved. What does this information suggest about the skills of pitchers in the past and the present. Write a brief report on your conclusions.

GRADE 8

ANSWER KEY FOR ACTIVITY 4

- ① a) There are only two possible outcomes in a game between two evenly matched teams: win and loss, and each outcome has a probability of 0.5. The toss of a fair coin also has only two equally likely possible outcomes: heads and tails. Therefore, we can simulate each game with the toss of a coin, and simulate a World Series with a best 4 out of 7 coin toss.
- b) The seven letters represent 7 games and the four H's represent four Blue Jay wins.

- ②. Answers will vary. However, the probability that the student writes in answer to this question should be the relative frequency of 7 tosses that shows in the student's table in Activity 2, exercise ⑦.

- ③ Answers will vary. However, the probability that the student writes in answer to this question should be the same as the student's answer to exercise ⑨c) in Activity 3.

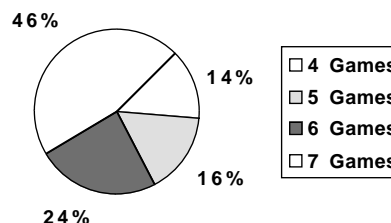
- ④. The answers to exercises ② and ③ should be close. If they are not close, the student may have made a computational error in determining the probability of a 7-game series in Activity 3, or the class simulation in Activity 2 had a n unusually large deviation from 5/16, attributable to chance. What is important here is that the student recognize that the answers should be close, and any deviation is a chance variation.

- ⑤ The completed table is shown below.

NUMBER OF GAMES IN THE WORLD SERIES 1947 – 1997				
	4 GAMES	5 GAMES	6 GAMES	7 GAMES
FREQUENCY	7	8	12	23
RELATIVE FREQUENCY	0.14	0.16	0.24	0.46

- ⑥. a) The relative frequencies of 4, 5, 6, and 7 games are respectively 0.14, 0.16, 0.24, and 0.46, as shown in the circle graph below.

Relative Frequencies of Each Number of Games



- ⑦. a) & b) The probability calculated in exercise ③ should be close to 5/16 or 0.3125, while the relative frequency in exercise ⑤ is 0.46. The student should realize that this might be a chance variation, or it may derive from other factors such as home team advantage or decreased motivation to win when up two games.

- ⑧. a) The *relative frequency* of an outcome is the fraction of times it has occurred, while the probability of an outcome is the estimated fraction of times that it is expected to occur.

b) The student explanation might include ideas such as, using a large number of trials, and ensuring randomness, or equal likelihood where the probability is computed on the assumption of equally likely outcomes.

- ⑨. a) The coin toss simulation assumes that both teams are evenly matched, and that the events are independent; that is, the outcome of any toss is not affected by the outcome of a previous toss.

b) If the teams are not evenly matched, then one team is better and this increases the chances that the series will end in 4 or 5 games rather than 6 or 7. That is, it *decreases* the chances that the World Series will end in 7 games.

INTERNET EXPLORATION

Students who access the websites given here will discover that the highest batting averages belonged to batters from the distant past, suggesting that the modern pitchers are significantly better than those of yesteryear.

The scoring guide presented below has been developed using student responses on a field test conducted in early 1998. However, it is recommended that you regard it merely as a starting point in the development of your own scoring guide that will evolve as you use this with students. *The Ontario Curriculum, Grades 1-8: Mathematics*, asserts,

Level 3, the “provincial standard”, identifies a high level of achievement of the provincial expectations. Parents of students achieving at Level 3 in a particular grade can be confident that their children will be prepared for work at the next grade.

For this reason, the scoring guides in this module shade the criteria in the Level 3 column, and on occasion reference other levels to Level 3 achievement.

Scoring Guide for Activity 4				
	Level 1	Level 2	Level 3	Level 4
CONCEPTS				
Interpretation of Simulations (exercises ❶ – ❹) DM 8-12, DM 8-15 DM 8-17, P 8-3, P 8-6, P 8-7	•Many answers incomplete. Explanations are vague and confused.	•Responses indicate confusion of the concepts, such as relative frequency and probability. •Little or no understanding of the purpose of a simulation	•Most responses to the exercises are correct. •Some evidence of an understanding of the effect of increasing the number of trials in a simulation	•Almost all responses are correct. •Clear understanding of the effect of increasing the number of trials in a simulation. •Gives clear explanations of inferences drawn.
COMMUNICATION				
Reporting of required knowledge related to concepts, procedures, & problem solving (exercises ❶ – ❹)	•Responses to exercises are usually incomplete and frequently incoherent.	•Responses to exercises are sometimes incomplete and often offer little or no explanation.	•Responses to exercises are mostly complete and usually presented clearly.	•Responses to exercises are complete and clearly presented, showing some creativity of content and expression.
COMMUNICATION				
Research: Information gleaned from electronic or print resources Internet Exploration DM 8-5	•Report shows little or no evidence of information gleaned from the Internet exploration.	•Report shows some evidence of information gleaned from the Internet exploration.	•Report shows significant evidence of information gleaned from the Internet exploration.	•In addition to Level 3 the report contains thoughtful inferences about the batting averages obtained from the websites.

WHAT YOU MIGHT SEE

REPORTING OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 1

This report addresses some of the questions, but several of the responses are difficult to interpret. For example, the response to exercise ④ does not provide any explanation. There is no response to exercise ⑦, and the response to exercise ⑨ is unclear. The report provides little evidence that the student understands the concepts involved.

WHAT YOU MIGHT SEE

REPORTING OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 3

The responses to the exercises are mostly complete. Some of the responses, such as ②, ③, and ④ are correct, but should have been expressed in sentence form, or clarified by adding a few words of explanation (See the Level 4 example on the next page). A clearly presented circle graph is given in the response to exercise ⑥, but no explanation is given.

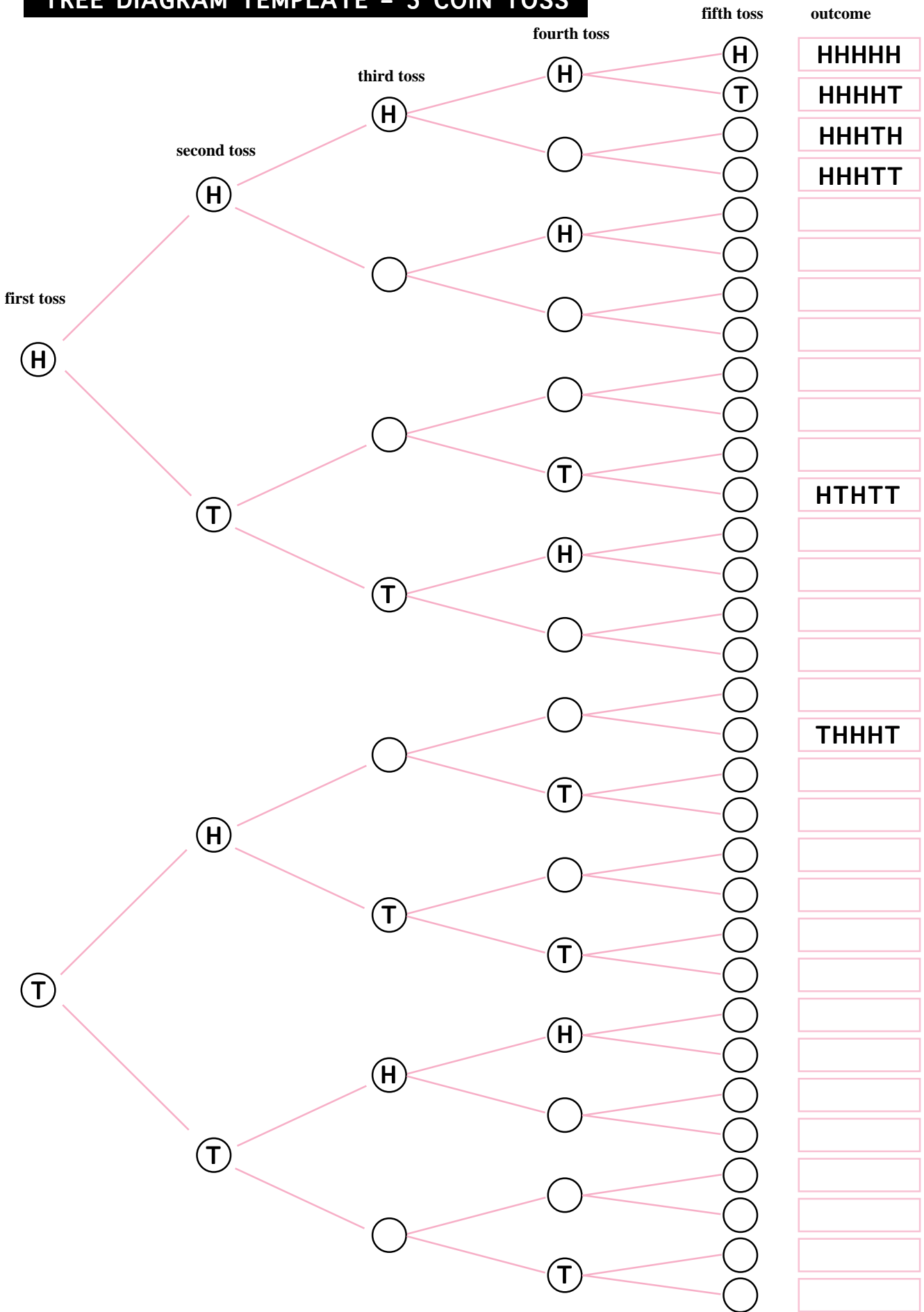
WHAT YOU MIGHT SEE

REPORTING OF REQUIRED KNOWLEDGE RELATED TO CONCEPTS, PROCEDURES & PROBLEM SOLVING

Level 4

The responses to the exercises are complete and clearly presented. The response to exercise ④ is exemplary, and shows an understanding of the concept of a simulation. In the response to exercise ⑦, the student indicates that there is a discrepancy between the predicted and observed frequencies of a 7-game World Series. Whether we agree with the inference drawn is irrelevant. The important issue is that the student understands and communicates the concepts well.

TREE DIAGRAM TEMPLATE – 5 COIN TOSS



Record of Student Achievement on the Grade 7 Unit

Will The African Elephant Become Extinct in Your Lifetime?

Student Name _____

From Scoring Guide for
Activity 1 p. 27

Topic	Level
Estimation	
Reporting	

From Scoring Guide for
Activity 2 p. 35

Topic	Level
Interpretation	
Bar Graphs	
Circle Graphs	

From Scoring Guide for
Activity 3 p. 43

Topic	Level
Problem Solving	
Stem-and-Leaf	
Mean, Median ..	

From Scoring Guide for
Activity 4 p. 51

Topic	Level
Reporting	
Research	

Combining the Marks
from all Scoring Guides

Summary	Level
Problem Solving	
Concepts	
Application	
Communication	

Student Name _____

From Scoring Guide for
Activity 1 p. 27

Topic	Level
Estimation	
Reporting	

From Scoring Guide for
Activity 2 p. 35

Topic	Level
Interpretation	
Bar Graphs	
Circle Graphs	

From Scoring Guide for
Activity 3 p. 43

Topic	Level
Problem Solving	
Stem-and-Leaf	
Mean, Median ..	

From Scoring Guide for
Activity 4 p. 51

Topic	Level
Reporting	
Research	

Combining the Marks
from all Scoring Guides

Summary	Level
Problem Solving	
Concepts	
Application	
Communication	

Student Name _____

From Scoring Guide for
Activity 1 p. 27

Topic	Level
Estimation	
Reporting	

From Scoring Guide for
Activity 2 p. 35

Topic	Level
Interpretation	
Bar Graphs	
Circle Graphs	

From Scoring Guide for
Activity 3 p. 43

Topic	Level
Problem Solving	
Stem-and-Leaf	
Mean, Median ..	

From Scoring Guide for
Activity 4 p. 51

Topic	Level
Reporting	
Research	

Combining the Marks
from all Scoring Guides

Summary	Level
Problem Solving	
Concepts	
Application	
Communication	

Record of Student Achievement on the Grade 8 Unit

Is the World Series Rigged?

Student Name _____

From Scoring Guide for
Activity 1 p. 65

Topic	Level
Problem Solving	
Simulation	

From Scoring Guide for
Activity 2 p. 73

Topic	Level
Histogram	
Circle Graph	

From Scoring Guide for
Activity 3 p. 81

Topic	Level
Problem Solving	
Tree Diagrams	

From Scoring Guide for
Activity 4 p. 89

Topic	Level
Simulations	
Reporting	
Research	

Combining the Marks
from all Scoring Guides

Summary	Level
Problem Solving	
Concepts	
Application	
Communication	

Student Name _____

From Scoring Guide for
Activity 1 p. 65

Topic	Level
Problem Solving	
Simulation	

From Scoring Guide for
Activity 2 p. 73

Topic	Level
Histogram	
Circle Graph	

From Scoring Guide for
Activity 3 p. 81

Topic	Level
Problem Solving	
Tree Diagrams	

From Scoring Guide for
Activity 4 p. 89

Topic	Level
Simulations	
Reporting	
Research	

Combining the Marks
from all Scoring Guides

Summary	Level
Problem Solving	
Concepts	
Application	
Communication	

Student Name _____

From Scoring Guide for
Activity 1 p. 65

Topic	Level
Problem Solving	
Simulation	

From Scoring Guide for
Activity 2 p. 73

Topic	Level
Histogram	
Circle Graph	

From Scoring Guide for
Activity 3 p. 81

Topic	Level
Problem Solving	
Tree Diagrams	

From Scoring Guide for
Activity 4 p. 89

Topic	Level
Simulations	
Reporting	
Research	

Combining the Marks
from all Scoring Guides

Summary	Level
Problem Solving	
Concepts	
Application	
Communication	

Additional Resources for Data Management & Probability

- Bloom, Stephen J. ed. *Data Exploration*, Focus Issue of *Teaching Children Mathematics*. Vol 2, #6. Reston, VA: National Council of Teachers of Mathematics, February, 1996.
- Burrill, Gail, ed. *Data Analysis*, Focus Issue of the *Mathematics Teacher*. Vol 83, #2. Reston, VA: National Council of Teachers of Mathematics, February, 1990.
- Corwin, Rebecca B, and Susan N. Friel. *Statistics: Prediction and Sampling*. from *Used Numbers: Real Data in the Classroom*. Palo Alto, CA: Dale Seymour Publications, 1990.
- Dixon, Juli K. and Christy J. Falba. *Graphing in the Information Age: Using Data from the World Wide Web*. *Mathematics Teaching in the Middle School*. Vol 2, #5. Reston, VA: National Council of Teachers of Mathematics, March-April, 1997. pp. 298-304.
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Free Software for Ontario Schools

The Ministry of Education and Training of Ontario purchases site licenses of software for all publically funded schools in the province. This software can be obtained from the Ontario Educational Software Service (OESS) representative in your school district. To determine what is available, access this web site: <http://www.tvo.org/osapac>

Errata Sheet

We are pleased to announce that the war against the gremlins that insert typos and other anomalies into the final document was won, but we must concede to them a couple of minor victories. These are itemized below.

- p. 44 The heading at the top of the page, below “WHAT YOU MIGHT SEE” should read, “Application of Mathematical Procedures: Constructed Stem-and-Leaf” .
- p. 71 & 72 The conversion of the histogram and circle graphs into EPS files resulted in the conversion of numeral 3 into the \$ symbol. Therefore, you should change the \$ symbol in the histogram on page 71 and the circle graph on page 72 into a 3 wherever it occurs.
- p. 76 In the fifth line under “Context”, the last outcome should read “TT” not “HT”.